

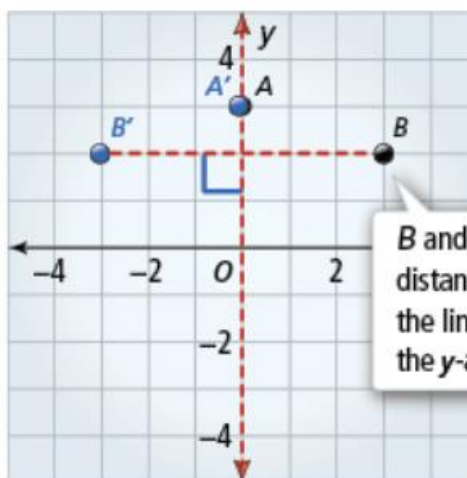
Content Area & Materials	Learning Objectives	Tasks	Check-in Opportunities	Submission of Work for Grades	
8th Grade Math PAPER PACKET: Digits 9-2 <ul style="list-style-type: none">• Lesson and examples• Close and Check• Homework worksheet Digits 9-3 <ul style="list-style-type: none">• Lesson and examples• Close and Check• Homework worksheet Digits 9-4 <ul style="list-style-type: none">• Lesson and examples• Close and Check• Homework worksheet ONLINE: <ul style="list-style-type: none">• Digits 9-2 (lessons and homework)• Digits 9-3 (lessons and homework)• Digits 9-4 (lessons and homework)	Essential Questions: What are the effects of the different transformations you can perform on 2D objects in the coordinate plane? Students will know... A reflection (flip) changes a figure’s position, not its size or shape. A rotation (turn) changes a figure’s position, not its size or shape.	PAPER PACKET with lesson, examples, “Close and Check,” and homework for Digits 9-2, 9-3, and 9-4. -or- ONLINE: Please log on to pearsonrealize.com to work through each part of the lessons for Digits 9-2, 9-3 and 9-4. The “Close and Check” page can be found by clicking on “Companion Page” at the bottom of the Close and Check screen for each lesson. Don’t forget to click on Solution at the bottom of each example and “Got it?” to check your answers.	Mrs. Wood is available during office hours at the times below by: <ul style="list-style-type: none">• Meeting on Microsoft Teams. Access by logging in with student email and password to Office 365 at https://www.tracy.k12.ca.us/students• by email (cwood@tusd.net)• call/text (209-597-8652) Email or call/text will get a response within 24 hours.	Students are expected to submit: <ol style="list-style-type: none">1. 9-2 Homework2. 9-3 Homework3. 9-4 Homework If submitting the PAPER PACKET, label with: Mrs. Wood Your full name class period ONLINE: Submit homework in Digits.	
Scheduled, if possible, Shared Experience	Teams meetings and phone calls can facilitate meaningful discussions.				
Scaffolds & Supports	Students working ONLINE should try out the Help functions in Digits. Notes for each lesson are included with the PAPER PACKETS.				
Teacher Office Hours Available by Teams, email, and call/text	Monday 10–11am	Tuesday 11:30am–12:30pm	Wednesday 10–11am	Thursday 11:30am–12:30pm	Friday 10–11am

Key Concept

A **reflection**, or flip, is a rigid motion that flips a figure over a line called the **line of reflection**.

If a point A is on the line of reflection, then its image A' is itself ($A' = A$).

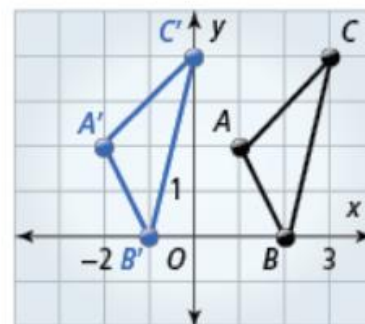
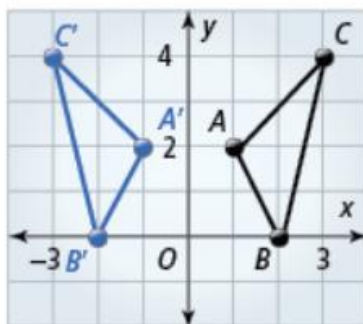
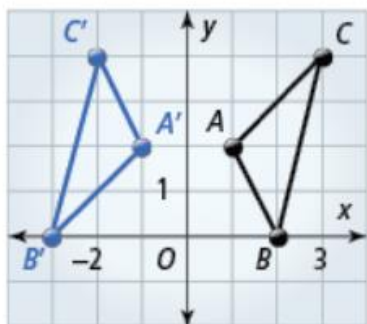
If a point B is not on the line of reflection, then B and B' are on opposite sides of the line of reflection. They are on a line perpendicular to the line of reflection, and are the same distance from the line of reflection.



Part 1

Example Recognizing Reflections

Determine which transformation is a reflection.

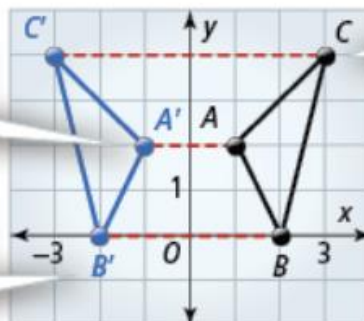


Solution

The second transformation is a reflection across the y-axis.

Since A is 1 unit to the right of the y-axis, A' is 1 unit to the left of the y-axis.

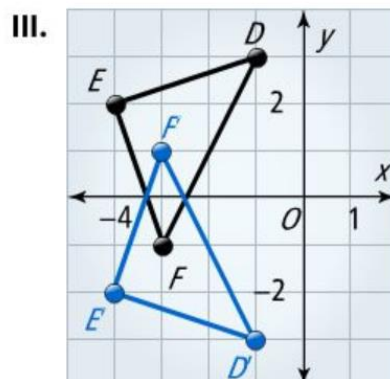
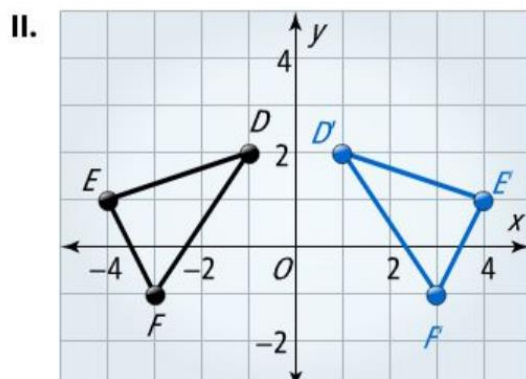
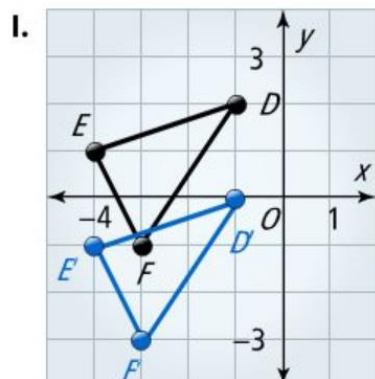
Since B is 2 units to the right of the y-axis, B' is 2 units to the left of the y-axis.



Since C is 3 units to the right of the y-axis, C' is 3 units to the left of the y-axis.

Got It?

Which graph shows a reflection of $\triangle DEF$ across the x-axis?



Part 2

Intro

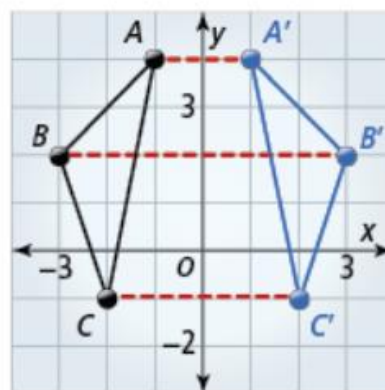
$\triangle A'B'C'$ is the image of $\triangle ABC$ after a reflection across the y-axis.

You can use arrow notation to show how each vertex of $\triangle ABC$ maps to its image after the reflection.

$$A(-1, 4) \longrightarrow A'(1, 4)$$

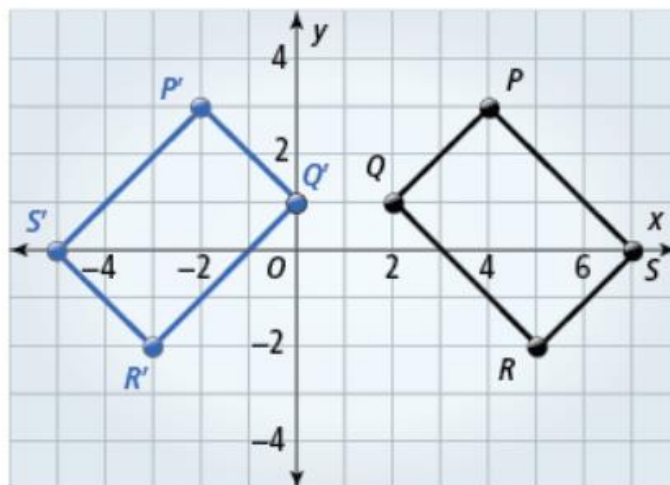
$$B(-3, 2) \longrightarrow B'(3, 2)$$

$$C(-2, -1) \longrightarrow C'(2, -1)$$



Example Describing Reflections

$PQRS$ is a rectangle. Describe in words how to map $PQRS$ to its image $P'Q'R'S'$. Then use arrow notation to show how each vertex of $PQRS$ maps to its image.



Solution

$P'Q'R'S'$ is the image of $PQRS$ after a reflection across the line $x = 1$.

P and P' are both 3 units from the line $x = 1$.

$$P(4, 3) \rightarrow P'(-2, 3)$$

S and S' are both 6 units from the line $x = 1$.

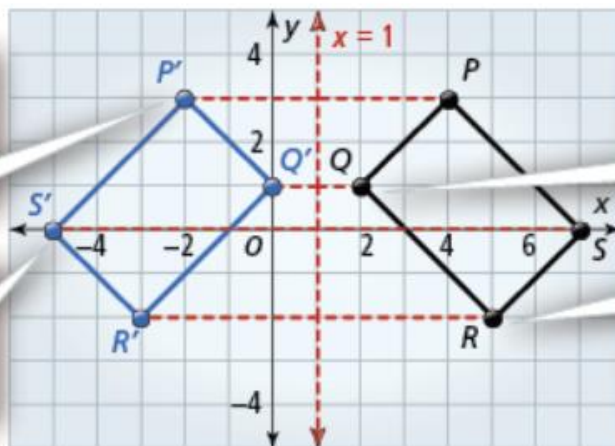
$$S(7, 0) \rightarrow S'(-5, 0)$$

Q and Q' are both 1 unit from the line $x = 1$.

$$Q(2, 1) \rightarrow Q'(0, 1)$$

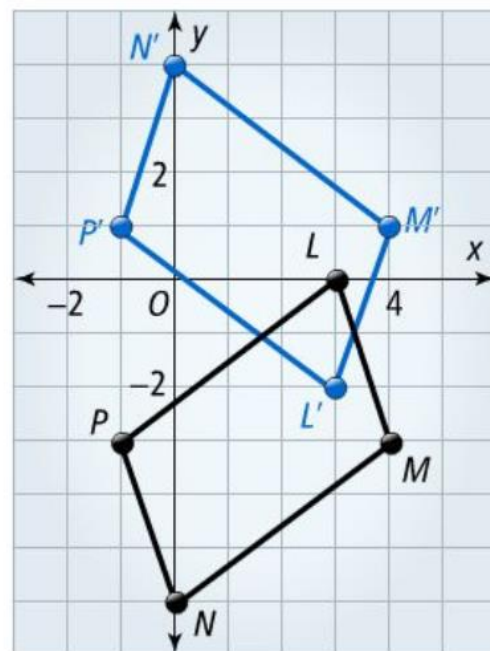
R and R' are both 4 units from the line $x = 1$.

$$R(5, -2) \rightarrow R'(-3, -2)$$



Got It?

$LMNP$ is a parallelogram. Describe in words how to map $LMNP$ to its image $L'M'N'P'$.



Part 3

Example Graphing Reflections

The vertices of $\triangle ABC$ are $A(1, 3)$, $B(-2, 4)$, and $C(-1, 1)$. Graph $\triangle ABC$ and $\triangle A'B'C'$, its image after a reflection across the x -axis.

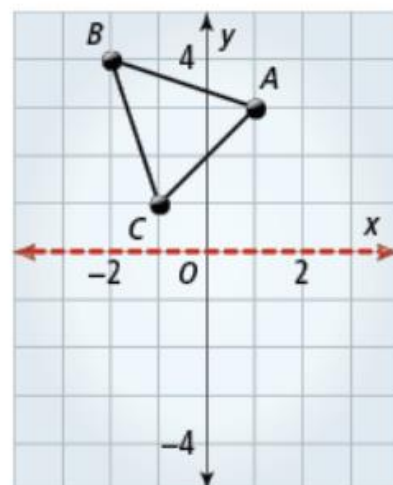
Solution

Step 1 Graph $\triangle ABC$. Show the x -axis as the line of reflection.

$A(1, 3)$

$B(-2, 4)$

$C(-1, 1)$

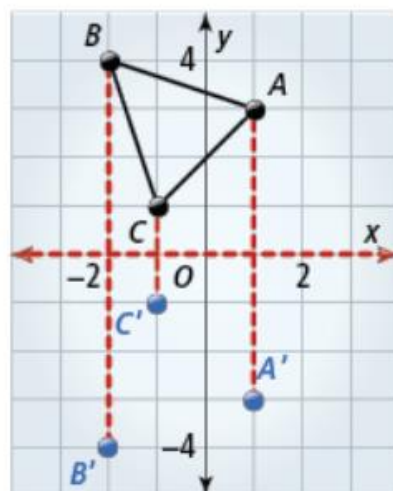


Step 2 Find the Image points A' , B' , and C' .

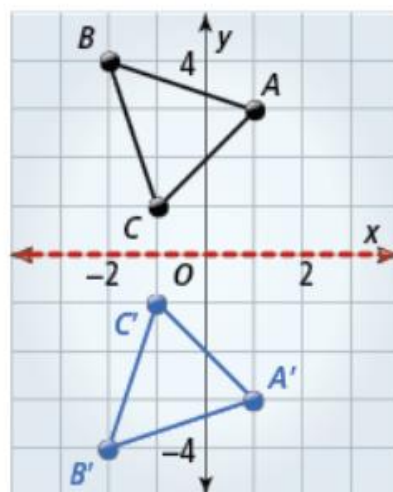
Since A is 3 units above the x -axis, A' is 3 units below the x -axis.

Since B is 4 units above the x -axis, B' is 4 units below the x -axis.

Since C is 1 unit above the x -axis, C' is 1 unit below the x -axis.

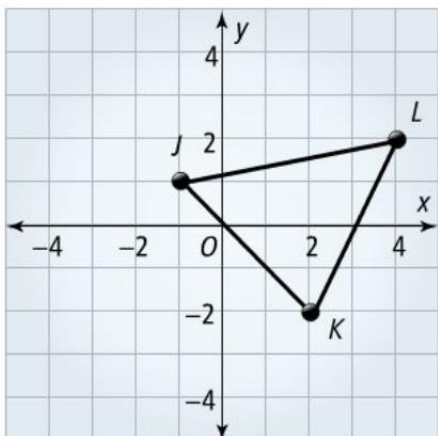


Step 3 Draw $\triangle A'B'C'$.



Got It?

If you reflect $\triangle JKL$ across the y -axis, what are the coordinates of J' ?



Part 3 (continued)

Got It?
Solutions

Part 1: Graph III

Part 2: $L'M'N'P'$ is the image of LMNP after a reflection across the line $y=-1$.

Part 3: (1, 1)

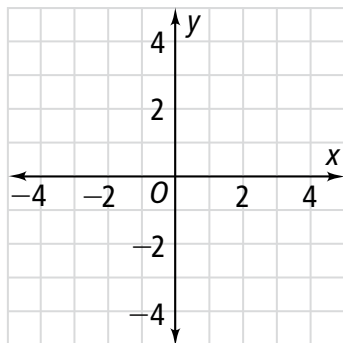
Close and Check

Focus Question

What effect does a flip have on a figure?

Do you know HOW?

1. The vertices of quadrilateral $QRST$ are $Q(-1, 3)$, $R(2, 2)$, $S(3, -2)$, $T(1, -2)$. Graph quadrilateral $QRST$ and quadrilateral $Q'R'S'T'$, its image after a reflection across the x -axis.



2. Use arrow notation to show how $QRST$ maps to $Q'R'S'T'$ from Exercise 1.

$Q(\text{ }) \rightarrow Q'(\text{ })$

$R(\text{ }) \rightarrow R'(\text{ })$

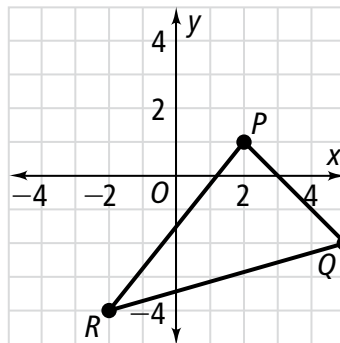
$S(\text{ }) \rightarrow S'(\text{ })$

$T(\text{ }) \rightarrow T'(\text{ })$

Do you UNDERSTAND?

3. **Compare and Contrast** How are translations and reflections the same and different?

4. **Error Analysis** A classmate says that the reflection across the x -axis of $\triangle PQR$ is $\triangle P'Q'R'$ where $P'(-2, 1)$, $Q'(-5, -2)$, and $R'(2, -4)$. What error did he make? What should the vertices be?



Close and Check

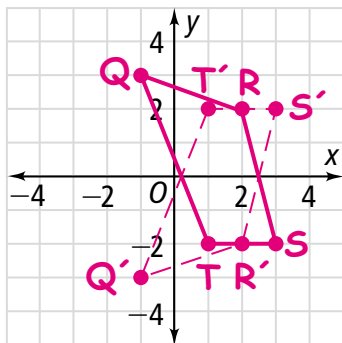
Focus Question

What effect does a flip have on a figure?

Sample: A flip changes only a figure's position, not its size or shape. The image of the figure faces the opposite direction of the figure.

Do you know HOW?

1. The vertices of quadrilateral $QRST$ are $Q(-1, 3)$, $R(2, 2)$, $S(3, -2)$, $T(1, -2)$. Graph quadrilateral $QRST$ and quadrilateral $Q'R'S'T'$, its image after a reflection across the x -axis.



2. Use arrow notation to show how $QRST$ maps to $Q'R'S'T'$ from Exercise 1.

$$Q(-1, 3) \rightarrow Q'(-1, -3)$$

$$R(2, 2) \rightarrow R'(2, -2)$$

$$S(3, -2) \rightarrow S'(3, 2)$$

$$T(1, -2) \rightarrow T'(1, 2)$$

SAMPLE SOLUTIONS ARE SHOWN BELOW.

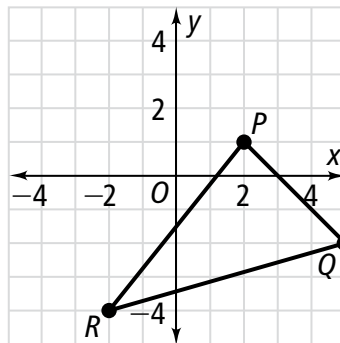
Do you UNDERSTAND?

3. **Compare and Contrast** How are translations and reflections the same and different?

Both maintain the size and shape of the original figure.

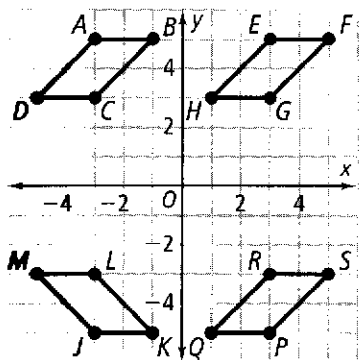
Translations maintain orientation. Reflections do not.

4. **Error Analysis** A classmate says that the reflection across the x -axis of $\triangle PQR$ is $\triangle P'Q'R'$ where $P'(-2, 1)$, $Q'(-5, -2)$, and $R'(2, -4)$. What error did he make? What should the vertices be?

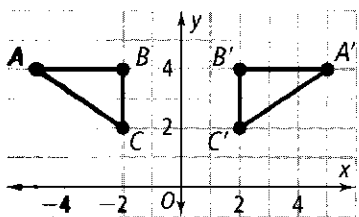


He reflected across the y -axis. $P'(2, -1)$, $Q'(5, 2)$, and $R'(-2, 4)$.

- The vertices of $\triangle ABC$ are $A(-5,4)$, $B(-2,4)$, and $C(-4,2)$. If $\triangle ABC$ is reflected across the y -axis to produce the image $\triangle A'B'C'$, find the coordinates of the vertex C' .
- The vertices of trapezoid $ABCD$ are $A(2,-2)$, $B(6,-2)$, $C(8,-7)$, and $D(1,-7)$. Draw a graph which shows $ABCD$ and $A'B'C'D'$ after a reflection across the y -axis.
- The vertices of $\triangle ABC$ are $A(-5,5)$, $B(-2,4)$, and $C(-2,3)$. Draw a graph which shows $\triangle ABC$ and its reflection across the x -axis, $\triangle A'B'C'$.
 - Graph the reflection of $\triangle A'B'C'$ across the y -axis.
- Writing** Which of the figures are reflections of the parallelogram $ABCD$?

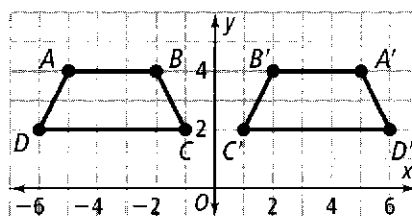


- Describe the reflections in words.
- Reasoning** One image of $\triangle ABC$ is $A'B'C'$.

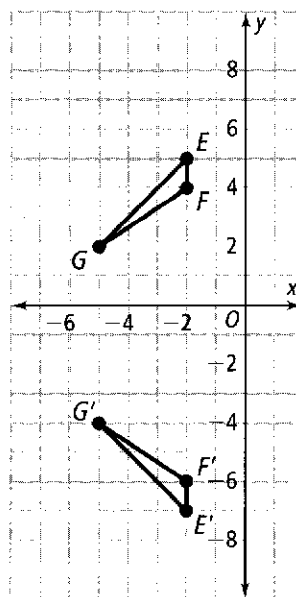


- How do the x -coordinates of the vertices change?
- How do the y -coordinates of the vertices change?
- What type of reflection is the image $\triangle A'B'C'$?

6. Think About the Process



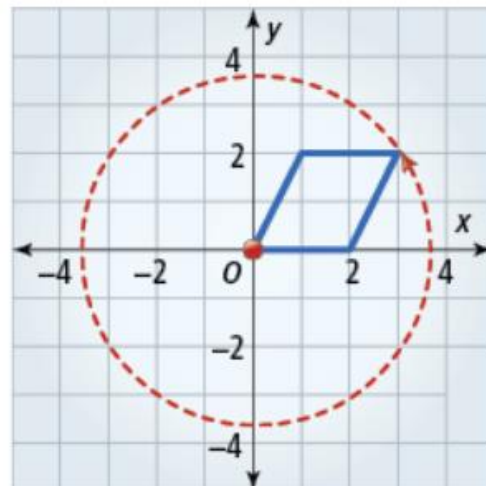
- What is true about a figure and an image created by a reflection? Select all that apply.
 - They are the same size.
 - The figure and the image are the same shape.
 - Each point on the image has the same x -coordinate as the corresponding point in the figure.
 - Each point on the image moves the same distance and direction from the figure.
 - One image of $ABCD$ is $A'B'C'D'$. What type of reflection is the image $A'B'C'D'$?
- Error Analysis** Your friend incorrectly says that the reflection of $\triangle EFG$ to its image $\triangle E'F'G'$ is a reflection across the x -axis.



- What is the correct description of the reflection?
- What is your friend's mistake?

Key Concept

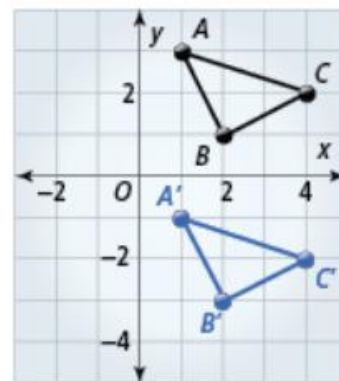
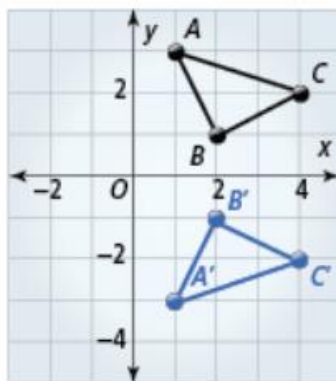
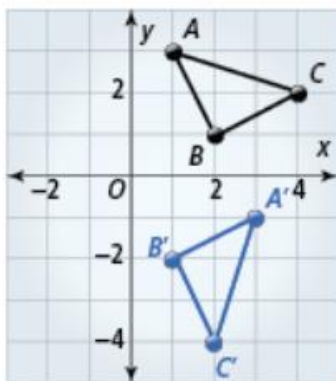
A **rotation** is a rigid motion that turns a figure about a fixed point called the **center of rotation**. The **angle of rotation** is the number of degrees the figure rotates. A positive angle of rotation turns the figure counterclockwise.



Part 1

Example Recognizing Rotations

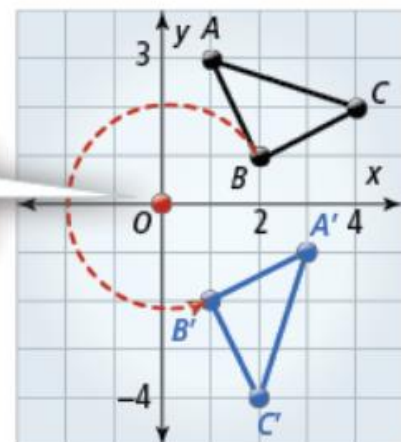
Identify which transformation is a rotation.



Solution

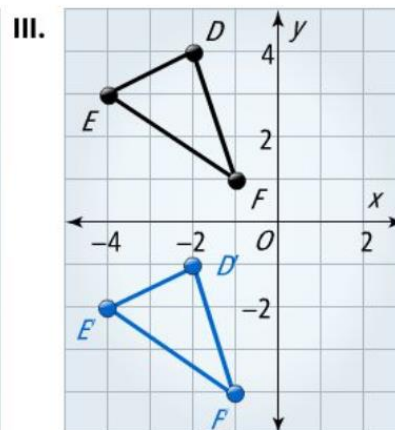
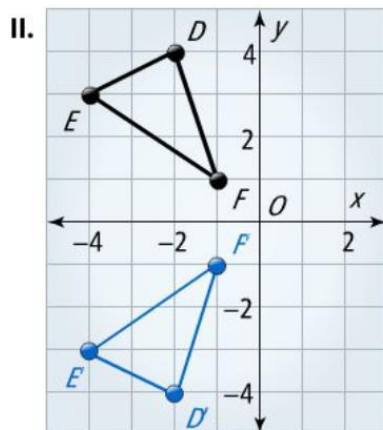
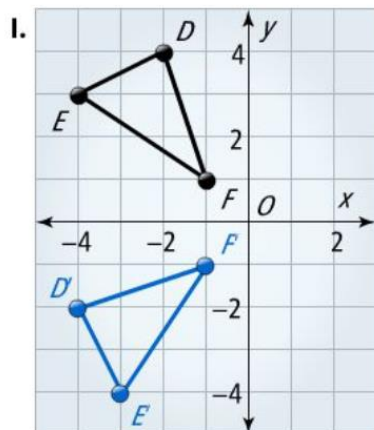
The first transformation is a rotation about the origin.

Center of rotation



Got It?

Which graph shows a rotation of $\triangle DEF$ about the origin?



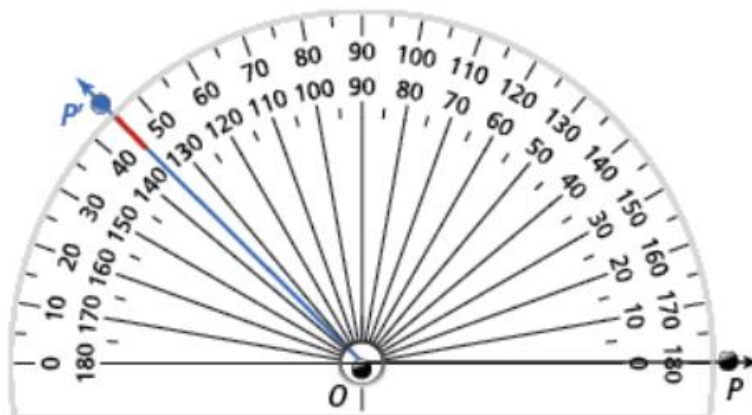
Part 2

Intro

You can use a protractor to find an angle of rotation.

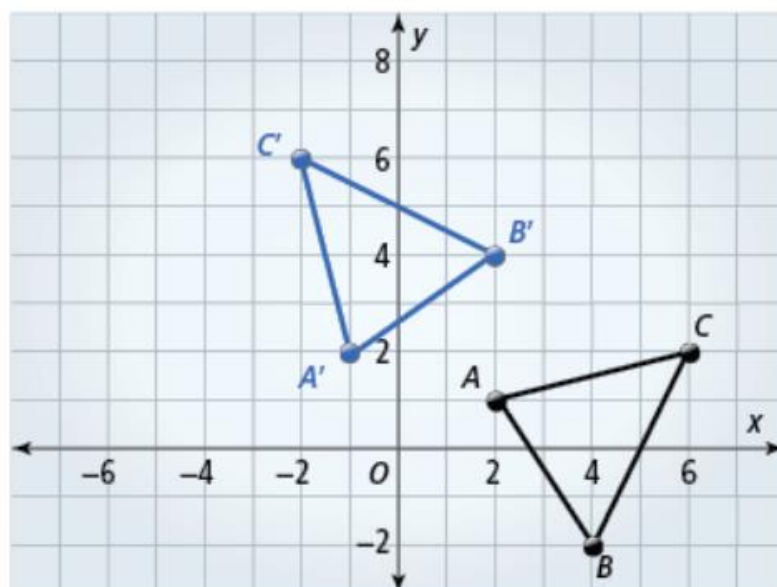
Suppose you have point P . You rotate point P about a center of rotation O .

The angle of rotation is 135° .



Example Finding Angles of Rotation

What is the angle of rotation about the origin that maps $\triangle ABC$ to $\triangle A'B'C'$?

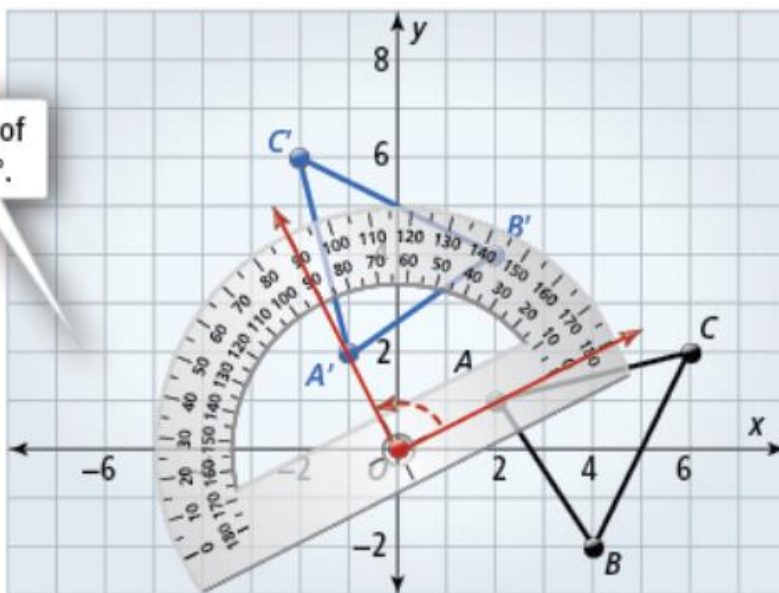


Solution

Draw and measure $\angle AOA'$.

The measure of $\angle AOA'$ is 90° .

The angle of rotation about the origin that maps $\triangle ABC$ to $\triangle A'B'C'$ is 90° .

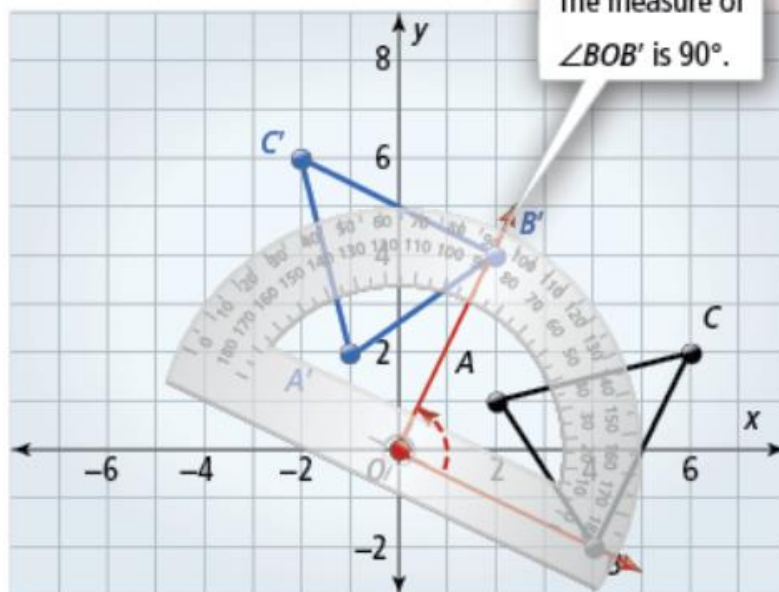


Draw and measure $\angle BOB'$.

The measure of $\angle BOB'$ equals the measure of $\angle AOA'$.

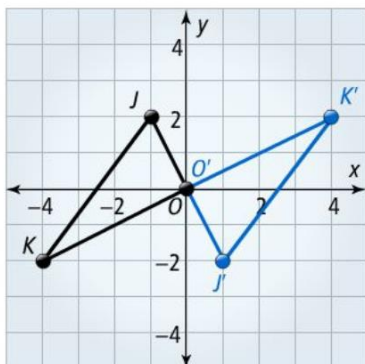
The answer checks. ✓

The measure of $\angle BOB'$ is 90° .



Got It?

What is the angle of rotation about the origin that maps $\triangle JKO$ to $\triangle J'K'O'$?



Part 3

Intro

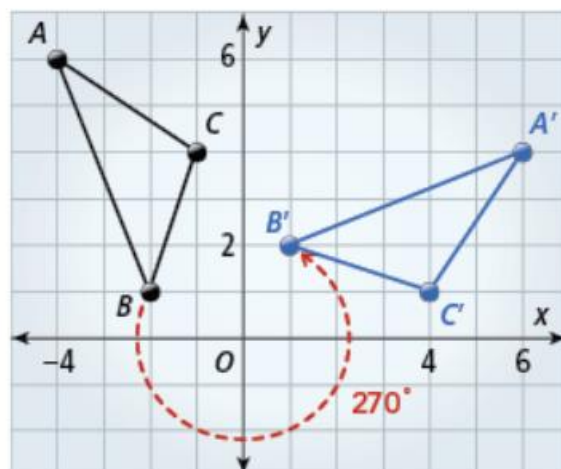
$\triangle A'B'C'$ is the image of $\triangle ABC$ after a 270° rotation about the origin.

You can use arrow notation to show how each vertex of $\triangle ABC$ maps to its image after the rotation.

$$A(-4, 6) \longrightarrow A'(6, 4)$$

$$B(-2, 1) \longrightarrow B'(1, 2)$$

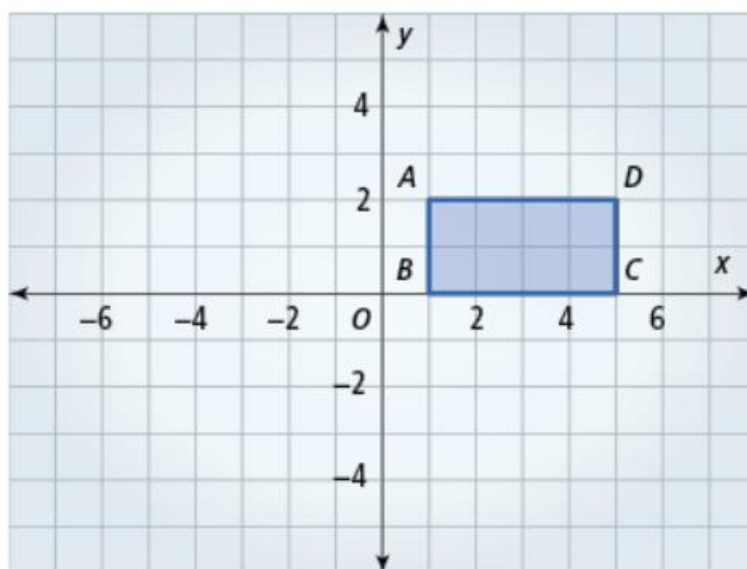
$$C(-1, 4) \longrightarrow C'(4, 1)$$



Example Graphing Rotations

Rectangle $ABCD$ has coordinates $A(1, 2)$, $B(1, 0)$, $C(5, 0)$, and $D(5, 2)$.

- Show the image of $ABCD$ after a rotation of 90° about the origin.
- Label the vertices of the image.
- Use arrow notation to show how each vertex of $ABCD$ maps to its image.



Solution

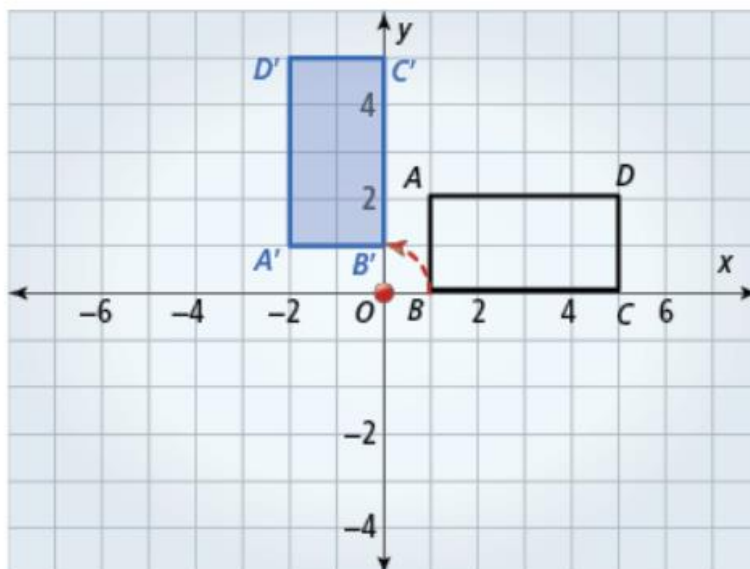
The blue rectangle $A'B'C'D'$ is the image of $ABCD$ after a rotation of 90° about the origin.


$$A(1, 2) \longrightarrow A'(-2, 1)$$

$$B(1, 0) \longrightarrow B'(0, 1)$$

$$C(5, 0) \longrightarrow C'(0, 5)$$

$$D(5, 2) \longrightarrow D'(-2, 5)$$



Part 3 (continued)	 Got It? Point P has coordinates $(3, 0)$. If you rotate P 270° about the origin, what are the coordinates of P' ?
Got It? Solutions	Part 1: Graph I Part 2: 180° Part 3: $(0, -3)$

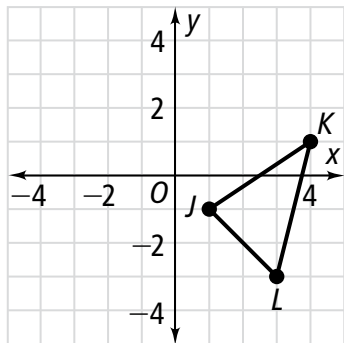
Close and Check

Focus Question

What effect does a turn have on a figure?

Do you know HOW?

1. Use arrow notation to show how $\triangle JKL$ maps to its image after a rotation 180° about the origin.



$J(\text{ }) \rightarrow J'(\text{ })$

$K(\text{ }) \rightarrow K'(\text{ })$

$L(\text{ }) \rightarrow L'(\text{ })$

2. The vertices of parallelogram $WXYZ$ are $W(-1, 1)$, $X(3, 2)$, $Y(3, -1)$, $Z(-1, -2)$. The vertices of its image, parallelogram $W'X'Y'Z'$, are $W'(-1, 1)$, $X'(3, 2)$, $Y'(3, -1)$, $Z'(-1, -2)$. What is the angle of rotation?

Do you UNDERSTAND?

3. **Compare and Contrast** How are reflections and rotations the same and different?

4. **Reasoning** Would the relationship between the vertices of any figure rotated 360° and its image always be true regardless of the point of rotation? Explain.

Close and Check

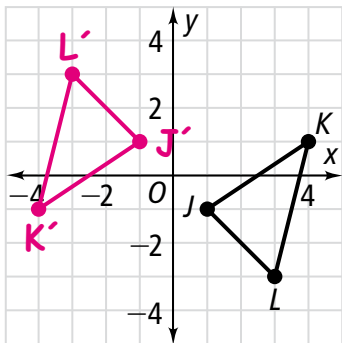
Focus Question

What effect does a turn have on a figure?

Sample: A turn changes only a figure's position, not its size or shape.

Do you know HOW?

1. Use arrow notation to show how $\triangle JKL$ maps to its image after a rotation 180° about the origin.



$$J(1, -1) \rightarrow J'(-1, 1)$$

$$K(4, 1) \rightarrow K'(-4, -1)$$

$$L(3, -3) \rightarrow L'(-3, 3)$$

2. The vertices of parallelogram $WXYZ$ are $W(-1, 1)$, $X(3, 2)$, $Y(3, -1)$, $Z(-1, -2)$. The vertices of its image, parallelogram $W'X'Y'Z'$, are $W'(-1, 1)$, $X'(3, 2)$, $Y'(3, -1)$, $Z'(-1, -2)$. What is the angle of rotation?

360°

SAMPLE SOLUTIONS ARE SHOWN BELOW.

Do you UNDERSTAND?

3. **Compare and Contrast** How are reflections and rotations the same and different?

Both change the orientation of the figure. Reflections flip the figure across a given line. Rotations turn the figure around a given point.

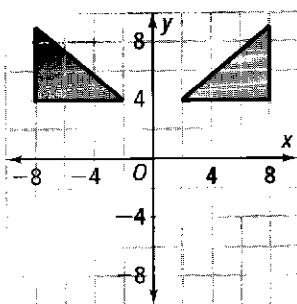
4. **Reasoning** Would the relationship between the vertices of any figure rotated 360° and its image always be true regardless of the point of rotation? Explain.

Yes, it does not matter if the point of rotation is inside, outside, or on the figure. The figure and its image will always be the same.

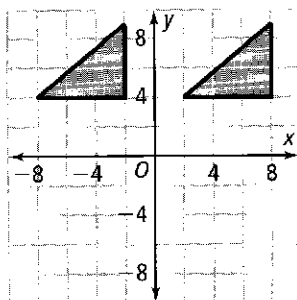


1. Which of these graphs shows a transformation that is a rotation?

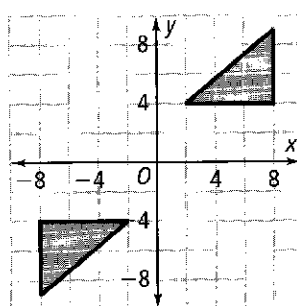
A.



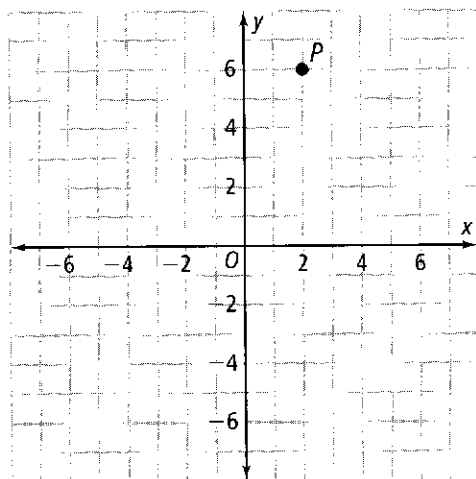
B.



C.

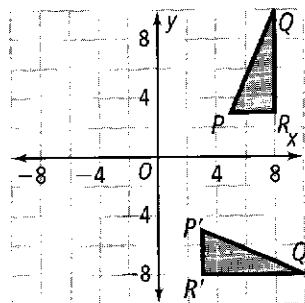


2. Point P has coordinates $(2, 6)$. If you rotate P 90° about the origin, $(0, 0)$, what are the coordinates of P' ?

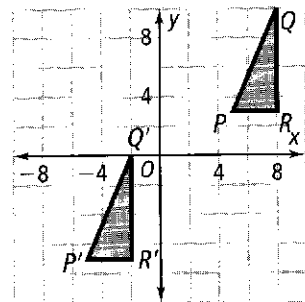


3. a. Which of these graphs shows a rotation of $\triangle PQR$ about the origin, $(0, 0)$?

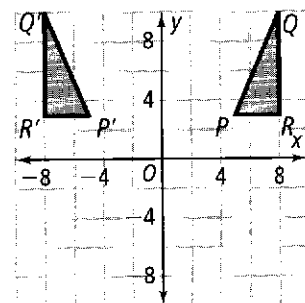
A.



B.



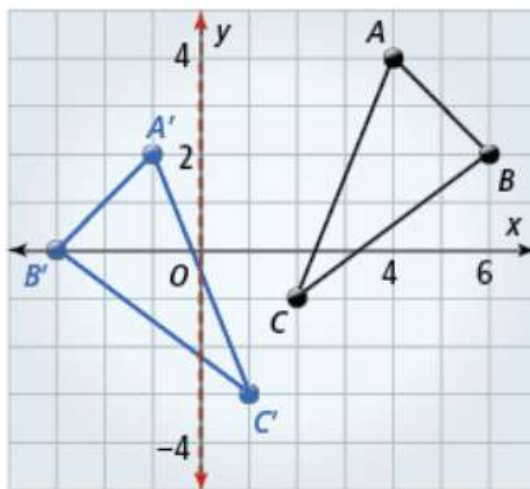
C.



- b. For each graph that does not show a rotation of $\triangle PQR$ about the origin, $(0, 0)$, describe what transformation the graph does show.

Key Concept

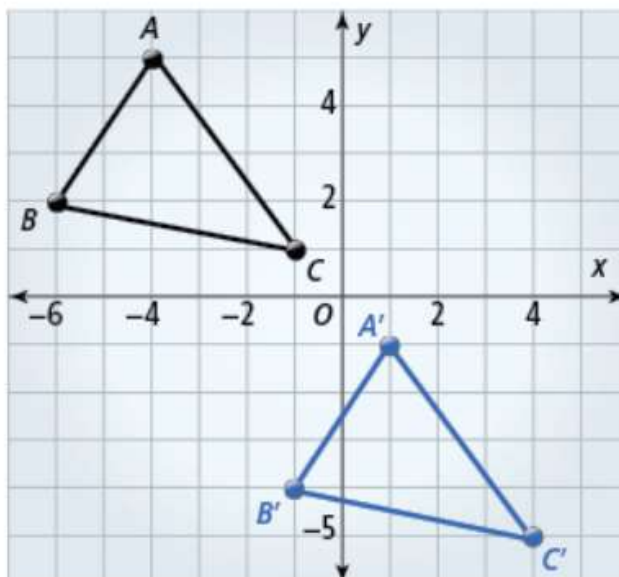
A two-dimensional figure is **congruent** to another two-dimensional figure if you can map one figure to the other by a sequence of rotations, reflections, and translations. The symbol \cong means "is congruent to."



Part 1

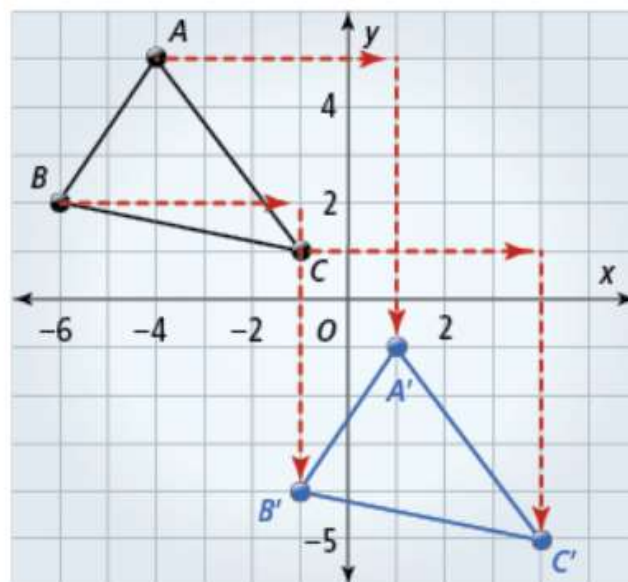
Example Describing Sequences of Translations

Given $\triangle ABC \cong \triangle A'B'C'$, describe a sequence of rigid motions that maps $\triangle ABC$ to $\triangle A'B'C'$.



Solution

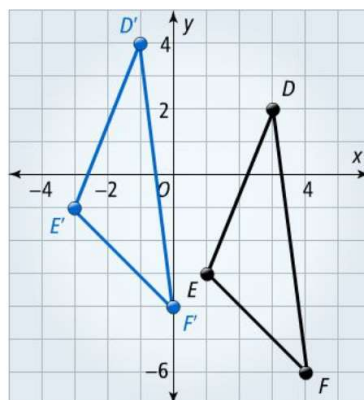
Method 1 Use a sequence of two translations. First, translate $\triangle ABC$ to the right 5 units. Then translate the triangle down 6 units. A translation of 5 units right followed by a translation of 6 units down maps $\triangle ABC$ to $\triangle A'B'C'$.



Method 2 Use a single translation. A single translation of 5 units right and 6 units down maps $\triangle ABC$ to $\triangle A'B'C'$.

Got It?

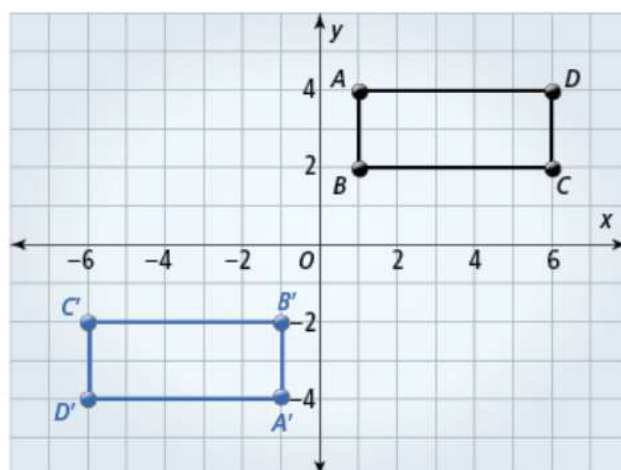
Given $\triangle DEF \cong \triangle D'E'F'$, describe a sequence of rigid motions that maps $\triangle DEF$ to $\triangle D'E'F'$.



Part 2

Example Describing Sequences of Rigid Motions

$ABCD$ is a rectangle. Given $ABCD \cong A'B'C'D'$, describe a sequence of rigid motions that maps $ABCD$ to $A'B'C'D'$.

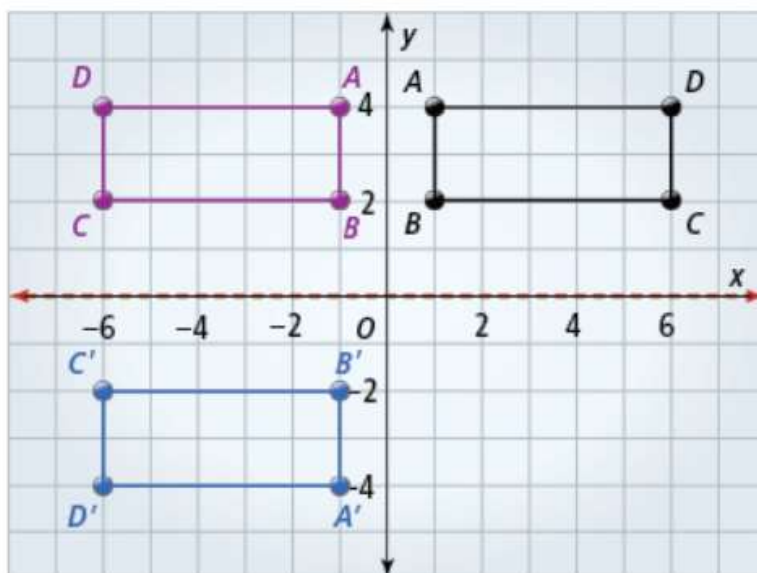


Solution

Method 1 Use two reflections.

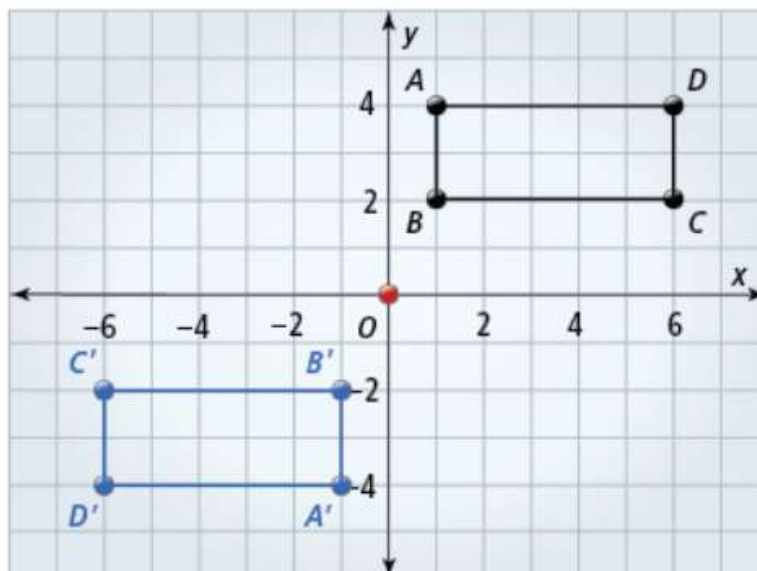
First, reflect $ABCD$ across the y -axis. Then reflect the image across the x -axis.

A reflection across the y -axis followed by a reflection across the x -axis maps $ABCD$ to $A'B'C'D'$.



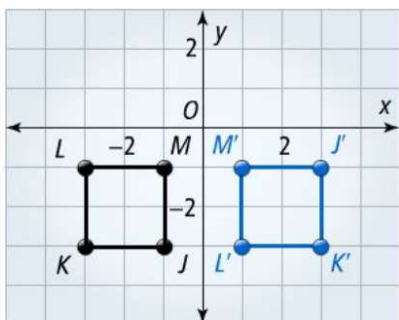
Method 2 Use a single rotation.

A single rotation of 180° about the origin maps $ABCD$ to $A'B'C'D'$.



Got It?

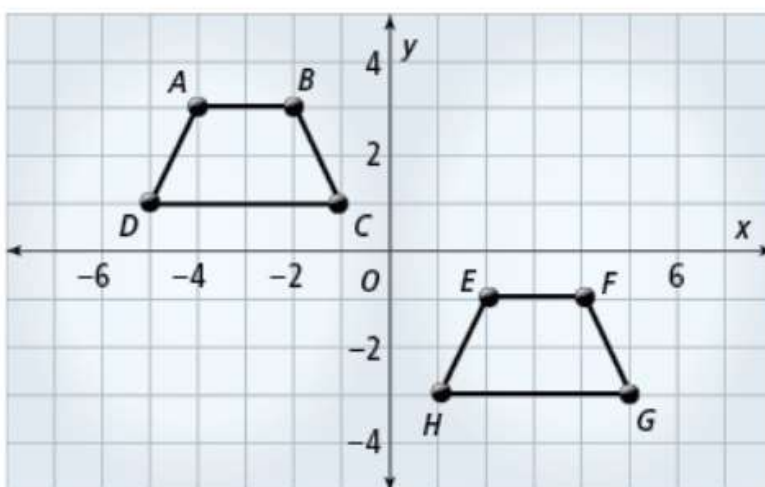
$JKLM$ is a square. Given $JKLM \cong J'K'L'M'$, describe a sequence of rigid motions that maps $JKLM$ to $J'K'L'M'$.



Part 3

Intro

Trapezoid $ABCD \cong$ Trapezoid $EFGH$



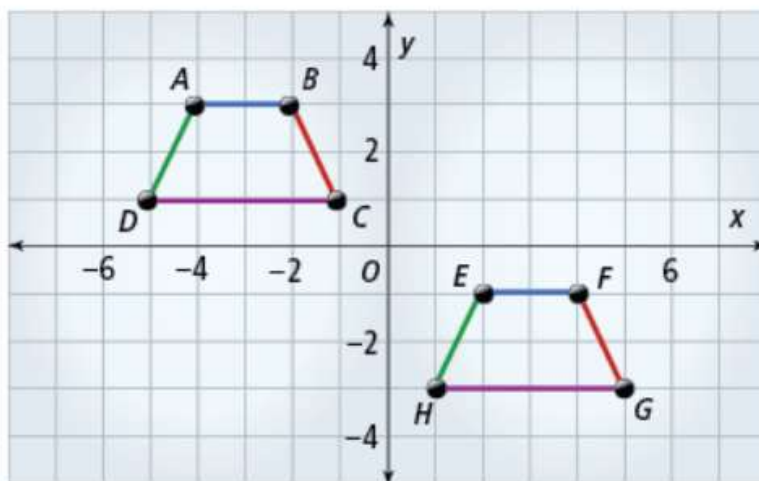
Effects on Line Segments Line segments are taken to line segments of the same length.

$$\overline{AB} \rightarrow \overline{EF}, \text{ so } \overline{AB} \cong \overline{EF}$$

$$\overline{BC} \rightarrow \overline{FG}, \text{ so } \overline{BC} \cong \overline{FG}$$

$$\overline{DC} \rightarrow \overline{HG}, \text{ so } \overline{DC} \cong \overline{HG}$$

$$\overline{AD} \rightarrow \overline{EH}, \text{ so } \overline{AD} \cong \overline{EH}$$



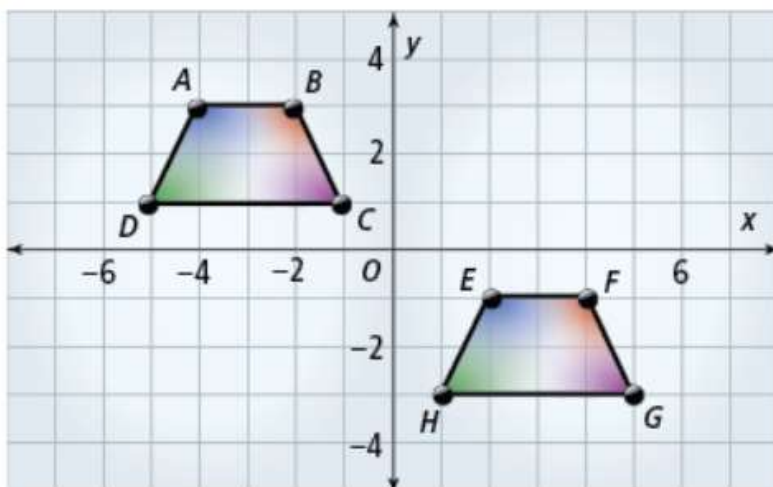
Effects on Angles Angles are taken to angles of the same measure.

$$\angle A \rightarrow \angle E, \text{ so } \angle A \cong \angle E$$

$$\angle B \rightarrow \angle F, \text{ so } \angle B \cong \angle F$$

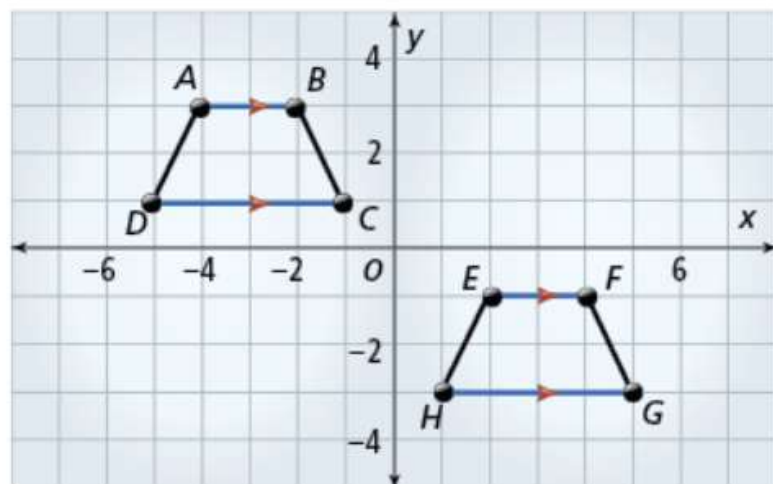
$$\angle C \rightarrow \angle G, \text{ so } \angle C \cong \angle G$$

$$\angle D \rightarrow \angle H, \text{ so } \angle D \cong \angle H$$



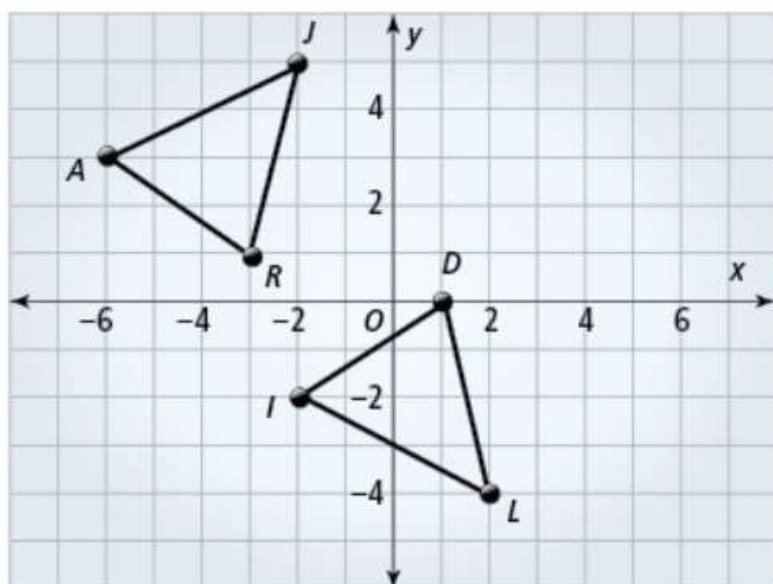
Effects on Parallel Lines Parallel lines are taken to parallel lines.

$$AB \parallel CD \rightarrow EF \parallel HG$$



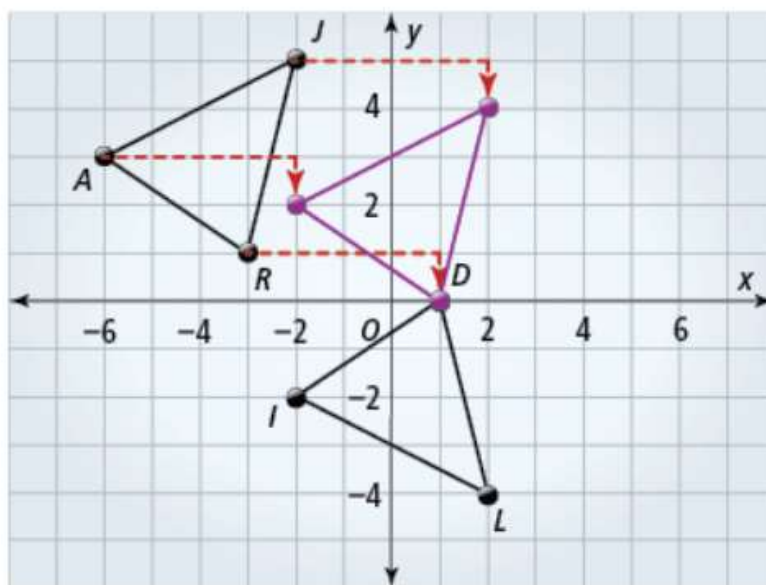
Example Determining If Figures are Congruent

Is $\triangle JAR \cong \triangle LID$? Explain.



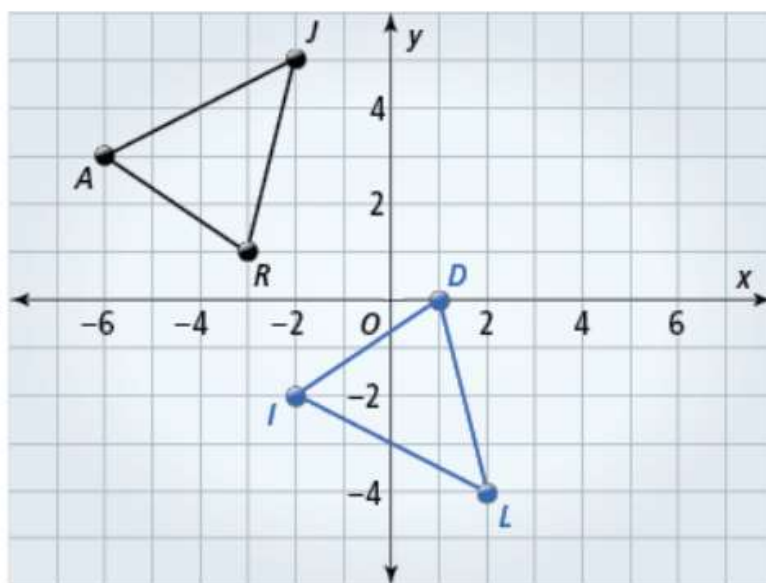
Solution

First, translate $\triangle JAR$ right 4 units and down 1 unit.



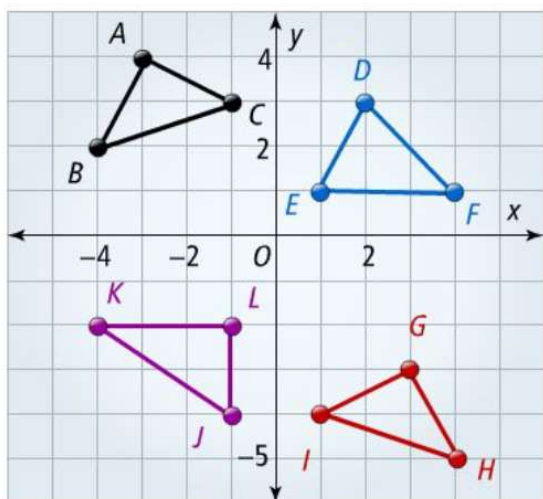
Next, reflect the image across the x -axis.

A translation of 4 units to the right and 1 unit down, followed by a reflection across the x -axis, maps $\triangle JAR$ to $\triangle LID$. So $\triangle JAR$ is congruent to $\triangle LID$.



Got It?

Which two triangles are congruent?



Got It? Solutions

Part 1: A translation of 4 units left followed by a translation of 2 units up maps $\triangle DEF$ to $\triangle D'E'F'$.

Part 2: A translation 4 units right, followed by a rotation of 90° about the point $(2, -2)$ maps $\triangle KLM$ to $\triangle K'L'M'$.

Part 3: $\triangle ABC \cong \triangle GHI$

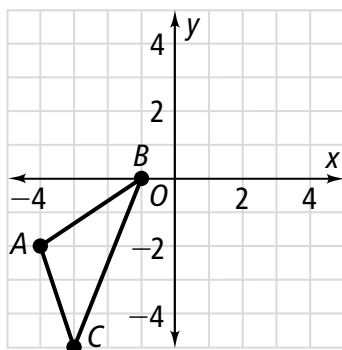
Close and Check

Focus Question

In what ways can you show that figures are identical?

Do you know HOW?

- Use arrow notation to show how $\triangle ABC$ maps to its image after a reflection across the x -axis followed by a reflection across the y -axis.



$A(\text{ }) \rightarrow A'(\text{ })$

$B(\text{ }) \rightarrow B'(\text{ })$

$C(\text{ }) \rightarrow C'(\text{ })$

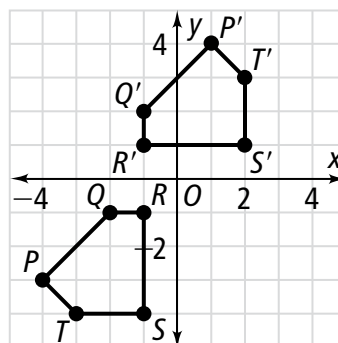
- What are the vertices of $\triangle DEF$, the image of $\triangle ABC$ above, after a reflection across the y -axis followed by a 180° rotation clockwise around the origin?

$D(\text{ }) \quad E(\text{ }) \quad F(\text{ })$

Do you UNDERSTAND?

- Reasoning** Assume $\triangle ABC$ in Problem 1 is rotated 180° about point B . What other transformation(s) could you use to map $\triangle ABC$ to $\triangle A'B'C'$?

- Writing** Describe a sequence of rigid motions that maps $PQRST$ to $P'Q'R'S'T'$.



Close and Check

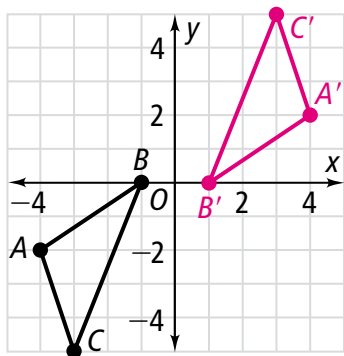
Focus Question

In what ways can you show that figures are identical?

Sample: To show that figures are identical, you can show that a sequence of rigid motions maps one figure to another.

Do you know HOW?

1. Use arrow notation to show how $\triangle ABC$ maps to its image after a reflection across the x-axis followed by a reflection across the y-axis.



$$A(-4, -2) \rightarrow A'(4, 2)$$

$$B(-1, 0) \rightarrow B'(1, 0)$$

$$C(-3, -5) \rightarrow C'(3, 5)$$

2. What are the vertices of $\triangle DEF$, the image of $\triangle ABC$ above, after a reflection across the y-axis followed by a 180° rotation clockwise around the origin?

$$D(-4, 2) \quad E(-1, 0) \quad F(-3, 5)$$

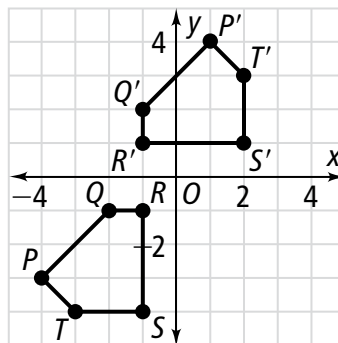
SAMPLE SOLUTIONS ARE SHOWN BELOW.

Do you UNDERSTAND?

3. **Reasoning** Assume $\triangle ABC$ in Problem 1 is rotated 180° about point B . What other transformation(s) could you use to map $\triangle ABC$ to $\triangle A'B'C'$?

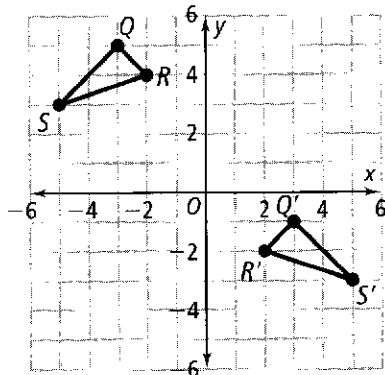
A reflection across the x-axis followed by a reflection across the line $x = -1$ results in the same image.

4. **Writing** Describe a sequence of rigid motions that maps $PQRST$ to $P'Q'R'S'T'$.



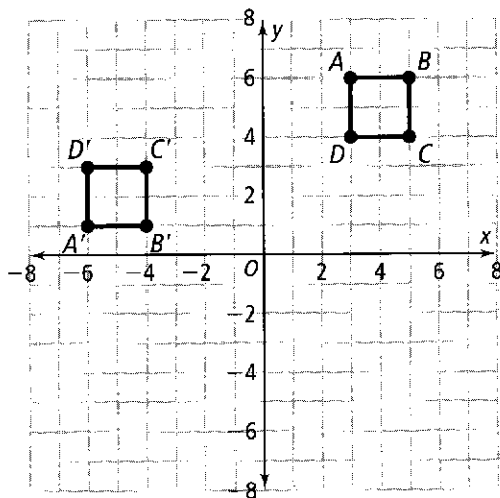
A 90° rotation about point R , a reflection across \overline{QR} , and a translation $\uparrow 2$ units results in figure $P'Q'R'S'T'$.

1. Given $\triangle QRS \cong \triangle Q'R'S'$, describe a pair of rigid motions that maps $\triangle QRS$ to $\triangle Q'R'S'$.

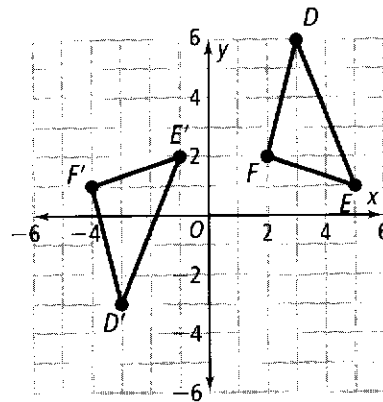


- A. reflection across the y -axis, translation of 6 units down
- B. rotation of 90° about the origin, translation of 6 units up
- C. reflection across the y -axis, translation of 10 units down
- D. translation of 10 units right, translation of 6 units down

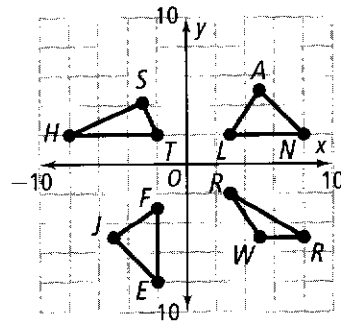
2. $ABCD$ is a square. Given $ABCD \cong A'B'C'D'$, describe a sequence of rigid motions that maps $ABCD$ to $A'B'C'D'$.



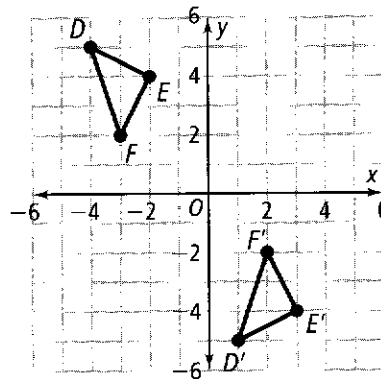
3. Is $\triangle DEF \cong \triangle D'E'F'$? Explain.



4. Which two triangles are congruent?



5. a. Writing Given $\triangle DEF \cong \triangle D'E'F'$, describe a pair of rigid motions that maps $\triangle DEF$ to $\triangle D'E'F'$.



- b. Describe a way you can show that $\triangle DEF$ is identical to $\triangle D'E'F'$.