LESSON 20.3

Applying the Zero Product Property to Solve Equations

Common Core Math Standards

The student is expected to:

CA CC **A-REI.4**

Solve quadratic equations in one variable. Also A-APR.3, A-SSE.2, A-SSE.3, F-LE.6

Mathematical Practices

CA CC **MP.2 Reasoning**

Language Objective

Explain what the Zero Product Property says, and give an example.

ENGAGE

Essential Question: How can you use the Zero Product Property to solve quadratic equations in factored form?

You can use the Zero Product Property to set each linear factor equal to 0 and then solve each resulting linear equation.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss why it might be a good idea for a pole vaulter to understand the shape of her path as she sails over the bar. Then preview the Lesson Performance Task.

Name ____________ Class ____________ Date ____________

20.3 Applying the Zero Product Property to Solve Equations

Essential Question: How can you use the Zero Product Property to solve quadratic equations in factored form?

Resource Locker

Explore Understanding the Zero Product Property

For all real numbers a and b, if the product of the two quantities equals zero, then at least one of the quantities equals zero.

Zero Product Property		
For all real numbers a and b, the following is true.		
Words	**Sample Numbers**	**Algebra**
If the product of two quantities equals zero, at least one of the quantities equals zero.	$9(\boxed{0}) = 0$ $0(4) = \boxed{0}$	If $ab = 0$, then $\boxed{a} = 0$ or $b = \boxed{0}$.

(A) Consider the equation $(x - 3)(x + 8) = 0$. Let $a = x - 3$ and $b = \boxed{x + 8}$.

(B) Since $ab = 0$, you know that $a = 0$ or $b = 0$. $\boxed{x - 3} = 0$ or $x + 8 = 0$

(C) Solve for x.

$x - 3 = 0$ or $x + 8 = 0$

$x = \boxed{3}$ $x = \boxed{-8}$

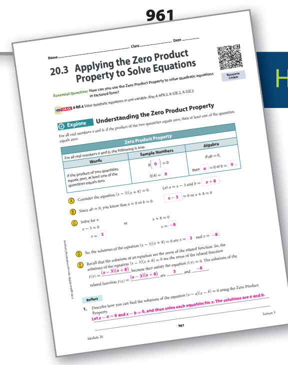
(D) So, the solutions of the equation $(x - 3)(x + 8) = 0$ are $x = \boxed{3}$ and $x = \boxed{-8}$.

(E) Recall that the solutions of an equation are the zeros of the related function. So, the solutions of the equation $(x - 3)(x + 8) = 0$ are the zeros of the related function $f(x) = \underline{(x - 3)(x + 8)}$ because they satisfy the equation $f(x) = 0$. The solutions of the related function $f(x) = \underline{(x - 3)(x + 8)}$ are $\underline{3}$ and $\underline{-8}$.

Reflect

1. Describe how you can find the solutions of the equation $(x - a)(x - b) = 0$ using the Zero Product Property.

Let $x - a = 0$ and $x - b = 0$, and then solve each equation for x. The solutions are a and b.



HARDCOVER PAGES 751–758

Turn to these pages to find this lesson in the hardcover student edition.

Explain 1 Applying the Zero Product Property to Functions

When given a function of the form $f(x) = (x + a)(x + b)$, you can use the Zero Product Property to find the zeros of the function.

Example 1 **Find the zeros of each function.**

(A) $f(x) = (x - 15)(x + 7)$

Set $f(x)$ equal to zero.	$(x - 15)(x + 7) = 0$		
Apply the Zero Product Property.	$x - 15 = 0$	or	$x + 7 = 0$
Solve each equation for x.	$x = 15$		$x = -7$

The zeros are 15 and -7.

(B) $f(x) = (x + 1)(x + 23)$

Set $f(x)$ equal to zero.	$(x + 1)(x + 23) = \boxed{0}$		
Apply the Zero Product Property.	$x + \boxed{1} = 0$	or	$x + 23 = \boxed{0}$
Solve for x.	$x = \boxed{-1}$		$x = \boxed{-23}$

The zeros are $\boxed{-1}$ and $\boxed{-23}$.

Reflect

2. **Discussion** Jordie was asked to identify the zeros of the function $f(x) = (x - 5)(x + 3)$. Her answers were $x = -5$ and $x = 3$. Do you agree or disagree? Explain.
Disagree; If $(x - 5)(x + 3) = 0$, then $x - 5 = 0$ or $x + 3 = 0$; so $x = 5$ or $x = -3$

3. How would you find the zeros of the function $f(x) = -4(x - 8)$?
The x-values that make $f(x) = 0$ are zeros of the function, so solve $-4(x - 8) = 0$. The only solution is 8.

4. What are the zeros of the function $f(x) = x(x - 12)$? Explain.
0 and 12 because they are the solutions of $x(x - 12) = 0$.

Your Turn

Find the zeros of each function.

5. $f(x) = (x - 10)(x - 6)$
$(x - 10)(x - 6) = 0$
$x - 10 = 0$, so $x = 10$, or
$x - 6 = 0$, so $x = 6$.
The zeros are 10 and 6.

6. $f(x) = 7(x - 13)(x + 12)$
$7(x - 13)(x + 12) = 0$
Since $7 \neq 0$, set only the factors $(x - 13)$ and $(x + 12)$ equal to 0 and solve.
$x - 13 = 0$, so $x = 13$, or $x + 12 = 0$, so $x = -12$.
The zeros are 13 and -12.

EXPLORE

Understanding the Zero Product Property

INTEGRATE TECHNOLOGY

To check their answers for the zeros of a function, students can enter the equation into a graphing calculator and use the **TABLE** function to find the x-values for which $y = 0$.

QUESTIONING STRATEGIES

If an equation has two factors, a and b, and $ab = 0$, can both a and b equal 0? Explain. **Yes; the Zero Product Property states that at least one of the factors a and b must equal zero. It is possible that both factors are equal to zero.**

EXPLAIN 1

Applying the Zero Product Property to Functions

AVOID COMMON ERRORS

When finding the zeros of a quadratic function, students sometimes choose the constants in the binomial factors as the solutions. For example, students might say that the zeros of $f(x) = (x - 2)(x - 9)$ are -2 and -9. Remind students to find the value of x that would make each factor equal 0. For this example, the zeros are 2 and 9.

QUESTIONING STRATEGIES

When using the Zero Product Property to find the zeros of a function, why do you set each factor equal to zero? **You set each factor equal to zero because when at least one of the factors is zero, the product is equal to zero.**

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practice **MP.2**, which calls for students to "reason abstractly and quantitatively." Students will rewrite quadratic equations so that they can apply the Zero Product Property to find the solutions. They build on their previous understanding of the connection between zeros, x-intercepts, and factors of quadratic functions, as well as the relationship between different forms of quadratic equations. In working with real-world problems, students interpret the solutions of equations to connect them to the real-world context.

EXPLAIN 2

Solving Quadratic Equations Using the Distributive Property and the Zero Product Property

AVOID COMMON ERRORS

When instructed to use the Distributive Property to rewrite an equation, students may think that they should expand the equation. Remind students that if they want to use the Zero Product Property to find the solutions to the equation, they should use the Distributive Property in the other direction, that is, to rewrite the equation as the product of factors.

QUESTIONING STRATEGIES

? What must be true about an equation for it to be possible to use the Distributive Property to rewrite that equation as the product of factors? **The same factor must appear in more than one term of the equation.**

? How can you verify that the solutions to an equation are correct? **You can substitute the values into the original equation to check that they make the equation true.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 For some equations, it is necessary to factor part of the equation before using the Distributive Property to rewrite the equation in factored form. Remind students to look for terms with a common factor.

Explain 2 Solving Quadratic Equations Using the Distributive Property and the Zero Product Property

The Distributive Property states that, for real numbers a, b, and c, $a(b+c) = ab + ac$ and $ab + ac = a(b+c)$. The Distributive Property applies to polynomials, as well. For instance, $3x(x-4) + 5(x-4) = (3x+5)(x-4)$. You can use the Distributive Property along with the Zero Product Property to solve certain equations.

Example 2 Solve each equation using the Distributive Property and the Zero Product Property.

(A) $3x(x-4) + 5(x-4) = 0$

Use the Distributive Property to rewrite the expression $3x(x-4) + 5(x-4)$ as a product. $3x(x-4) + 5(x-4) = (3x+5)(x-4)$

Rewrite the equation. $(3x+5)(x-4) = 0$

Apply the Zero Product Property. $3x + 5 = 0$ or $x - 4 = 0$

Solve each equation for x. $3x = -5$, $x = -\frac{5}{3}$; $x = 4$

The solutions are $x = -\frac{5}{3}$ and $x = 4$.

(B) $-9(x+2) + 3x(x+2) = 0$

Use the Distributive Property to rewrite the expression $-9(x+2) + 3x(x+2)$ as a product. $-9(x+2) + 3x(x+2) = (\boxed{-9} + 3x)(x + \boxed{2})$

Rewrite the equation. $(\boxed{-9} + 3x)(x + \boxed{2}) = 0$

Apply the Zero Product Property. $\boxed{-9} + 3x = 0$ or $x + \boxed{2} = 0$

Solve each equation for x. $3x = \boxed{9}$, $x = \boxed{3}$; $x = \boxed{-2}$

The solutions are $x = \boxed{3}$ and $x = \boxed{-2}$.

Reflect

7. How can you solve the equation $5x(x-3) + 4x - 12 = 0$ using the Distributive Property?
Factor $4x - 12$ as $4(x-3)$ first. Then $5x(x-3) + 4(x-3) = 0$ becomes $(5x+4)(x-3) = 0$.

Your Turn

Solve each equation using the Distributive Property and the Zero Product Property.

8. $7x(x-11) - 2(x-11) = 0$
$(7x-2)(x-11) = 0$
$7x - 2 = 0$ or $x - 11 = 0$
$x = \frac{2}{7}$ $\quad x = 11$

9. $-8x(x+6) + 3x + 18 = 0$
$-8x(x+6) + 3(x+6) = 0$
$(-8x+3)(x+6) = 0$
$-8x + 3 = 0$ or $x + 6 = 0$
$x = \frac{3}{8}$ $\quad x = -6$

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have students work in pairs. Have each student write two factored equations: one in the form $y = k(x-a)(x-b)$, and one that is factored so that the x terms have coefficients other than 1. Students solve their equations, then trade equations and solve both equations written by their partners. Have partners discuss how they solved each equation and tell which equations they found easiest to solve.

Explain 3 Solving Real-World Problems Using the Zero Product Property

Example 3

The height of one diver above the water during a dive can be modeled by the equation $h = -4(4t + 5)(t - 3)$, where h is height in feet and t is time in seconds. Find the time it takes for the diver to reach the water.

Analyze Information

Identify the important information.

- The height of the diver is given by the equation $h = -4(4t + 5)(t - 3)$.
- The diver reaches the water when $h = \boxed{0}$.

Formulate a Plan

To find the time it takes for the diver to reach the water, set the equation equal to $\boxed{0}$ and use the Zero Product Property to solve for t.

Solve

Set the equation equal to zero. $-4(4t + 5)(t - 3) = 0$

Apply the Zero Product Property. $4t + 5 = 0$ or $t - 3 = 0$

Since $-4 \neq 0$, set the other factors equal to 0.

Solve each equation for x.

$4t + 5 = 0$ or $t - 3 = 0$

$4t = \boxed{-5}$ $\quad t = \boxed{3}$

$t = \boxed{-\frac{5}{4}}$

The zeros are $t = \boxed{-\frac{5}{4}}$ and $t = \boxed{3}$. Since time cannot be negative, the time it takes for the diver to reach the the water is $\boxed{3}$ seconds.

Justify and Evaluate

Check to see that the answer is reasonable by substituting 3 for t in the equation $-4(4t + 5)(t - 3) = 0$.

$$-4\big(4(3) + 5\big)\big((3) - 3\big) = -4\left(\boxed{12} + 5\right)\left(\boxed{3} - 3\right)$$

$$= -4\left(\boxed{17}\right)\left(\boxed{0}\right)$$

$$= \boxed{0}$$

Since the equation is equal to $\boxed{0}$ for $t = 3$, the solution is reasonable. The diver will reach the water after $\boxed{3}$ seconds.

EXPLAIN 3

Solving Real-World Problems Using the Zero Product Property

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 When working with real-world problems that give the factored form of a quadratic function modeling the height of a falling object, have students multiply the factors and write the function in standard form. They should find that all the functions fit the same standard form: $f(t) = -16t^2 + v_0t + h_0$, where $f(t)$ is the height in feet at time t, t is the time in seconds, v_0 is the initial vertical velocity, and h_0 is the initial height. (The coefficient of t^2 will differ if the units are different.) Explain that this general form is found in all problems modeling falling objects because the force of gravity affects the motion of all objects in the same way.

DIFFERENTIATE INSTRUCTION

Multiple Representations

For problems involving quadratic functions that model the height of a falling object, have students create a table of values for the height of the object. Continue the table until the object reaches a height of 0. Point out to students that the time when $h = 0$ represents the time it takes for the object to reach the ground. Discuss the differences between using this method and using the Zero Product Property to find the solution to a falling object problem.

QUESTIONING STRATEGIES

Not all quadratic functions can be written in a factored form. Can quadratic functions that model the height of a falling object always be written in factored form? Explain. **Yes; a function modeling the height of a falling object will have a value of 0 when the object reaches the ground, so it will have *x*-intercepts. Any quadratic function that has *x*-intercepts can be written in factored form.**

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Discuss with students whether the Zero Product Property can be applied only to quadratic functions. Students should realize that the Zero Product Property can be used to find the x-intercept of any factorable function.

SUMMARIZE THE LESSON

How do you solve a quadratic equation using the Zero Product Property? **If the function is in the form $f(x) = (x + a)(x + b)$, you can use the Zero Product Property by setting both $x + a = 0$ and $x + b = 0$, and finding both solutions for x.**

Reflect

10. If you were to graph the function $f(t) = -4(4t + 5)(t - 3)$, what points would be associated with the zeros of the function?
the points associated with the *x*-intercepts of the graph of the function, $\left(-\frac{5}{4}, 0\right)$ and $(3, 0)$

Your Turn

11. The height of a golf ball after it has been hit from the top of a hill can be modeled by the equation $h = -8(2t - 4)(t + 1)$, where h is height in feet and t is time in seconds. How long is the ball in the air?

The ball is in the air from the time it leaves the ground to the time it returns to the ground.

The height of the ball when it returns to the ground is $h = 0$.

Set the equation equal to 0.

$-8(2t - 4)(t + 1) = 0$

Since $-8 \neq 0$, set the factors $(2t - 4)$ and $(t + 1)$ equal to 0 and solve each equation.

$2t - 4 = 0$, so $t = 2$, or $t + 1 = 0$, so $t = -1$.

The zeros are $t = 2$ and $t = -1$. Since time cannot be negative, the ball is in the air for 2 seconds.

Elaborate

12. Can you use the Zero Product Property to find the zeros of the function $f(x) = (x - 1) + (2 - 9x)$? Explain.
No; the expression $(x - 1) + (2 - 9x)$ is not a product of factors, so the Zero Product Property cannot be used.

13. Suppose a and b are the zeros of a function. Name two points on the graph of the function and explain how you know they are on the graph. What are the x-coordinates of the points called?
$(a, 0)$ and $(b, 0)$; zeros of a function are values of x that make the function value 0; the *x*-intercepts of the graph of the function.

14. **Essential Question Check-In** Suppose you are given a quadratic function in factored form that is set equal to 0. Why can you solve it by setting each factor equal to 0?
The Zero Product Property states that if the product of two numbers is 0, then at least one of the numbers must be 0. A quadratic function in factored form is the product of two linear factors, so at least one of the factors must be 0.

LANGUAGE SUPPORT EL

Connect Vocabulary

When encountering the Zero Product Property for the first time, the formal construction of the sentences in the property may be difficult for English learners to understand. Point out that the phrase *the following is true* directs the reader's attention to the information that follows and emphasizes that it is true.