LESSON 19.3

Interpreting Vertex Form and Standard Form

Common Core Math Standards

The student is expected to:

CA CC **F-IF.4**

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Also F-IF.8, F-IF.2, F-IF.4, F-BF.1

Mathematical Practices

CA CC **MP.2 Reasoning**

Language Objective

Work with a partner to describe how to write quadratic functions in vertex form and standard form.

ENGAGE

Essential Question: How can you change the vertex form of a quadratic function to standard form?

Sample answer: You can rewrite the quadratic expression in the vertex form by multiplying and then combining like terms so that the function rule is written in descending order of the exponents.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo and the type of data that is needed in order to represent the path of the tennis ball as a function. Then preview the Lesson Performance Task.

Name ______________________ Class ____________ Date ________

19.3 Interpreting Vertex Form and Standard Form

Essential Question: How can you change the vertex form of a quadratic function to standard form?

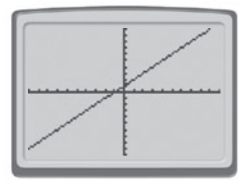
Resource Locker

Explore Identifying Quadratic Functions from Their Graphs

Determine whether a function is a quadratic function by looking at its graph. If the graph of a function is a parabola, then the function is a quadratic function. If the graph of a function is not a parabola, then the function is not a quadratic function.

Use a graphing calculator to graph each of the functions. Set the viewing window to show −10 to 10 on both axes. Determine whether each function is a quadratic function.

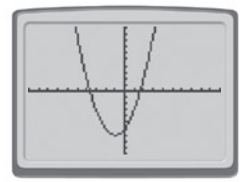
(A) Use a graphing calculator to graph $f(x) = x + 1$.



(B) Determine whether the function $f(x) = x + 1$ is a quadratic function.

The function $f(x) = x + 1$ **is not** a quadratic function.

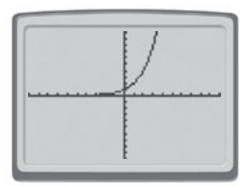
(C) Use a graphing calculator to graph $f(x) = x^2 + 2x - 6$.



(D) Determine whether the function $f(x) = x^2 + 2x - 6$ is a quadratic function.

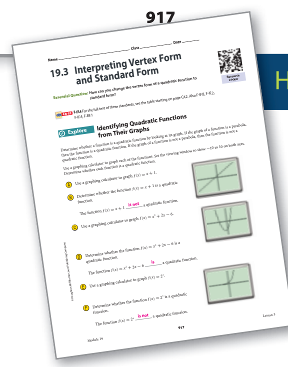
The function $f(x) = x^2 + 2x - 6$ **is** a quadratic function.

(E) Use a graphing calculator to graph $f(x) = 2^x$.



(F) Determine whether the function $f(x) = 2^x$ is a quadratic function.

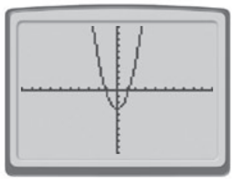
The function $f(x) = 2^x$ **is not** a quadratic function.



HARDCOVER PAGES 717–728

Turn to these pages to find this lesson in the hardcover student edition.

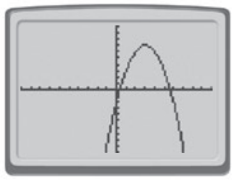
Ⓖ Use a graphing calculator to graph $f(x) = 2x^2 - 3$.



Ⓗ Determine whether the function $f(x) = 2x^2 - 3$ is a quadratic function.

The function $f(x) = 2x^2 - 3$ **is** a quadratic function.

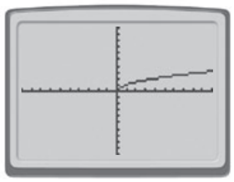
Ⓘ Use a graphing calculator to graph $f(x) = -(x - 3)^2 + 7$.



Ⓙ Determine whether the function $f(x) = -(x - 3)^2 + 7$ is a quadratic function.

The function $f(x) = -(x - 3)^2 + 7$ **is** a quadratic function.

Ⓚ Use a graphing calculator to graph $f(x) = \sqrt{x}$.



Ⓛ Determine whether the function $f(x) = \sqrt{x}$ is a quadratic function.

The function $f(x) = \sqrt{x}$ **is not** a quadratic function.

Reflect

1. How can you determine whether a function is quadratic or not by looking at its graph?
Sample answer: All quadratic functions have graphs that are parabolas, opening either upward or downward. If the graph of a function does not have this shape, then the function is not quadratic.

2. **Discussion** How can you tell if a function is a quadratic function by looking at the equation?
Sample answer: If the highest power of the variable is x^2, then the function is quadratic.

EXPLORE

Identifying Quadratic Functions from Their Graphs

INTEGRATE TECHNOLOGY

Students have the option of completing the activity either in the book or online.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Patterns

MP.8 Point out to students that the graph of a parabola is symmetric about a vertical line through its vertex.

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practice **MP.2,** which calls for students to “reason abstractly and quantitatively.” Students analyze the relationship between quadratic functions in standard and vertex forms and convert between vertex form and standard form.

EXPLAIN 1

Identifying Quadratic Functions in Standard Form

QUESTIONING STRATEGIES

How can you tell by looking at an equation whether the value of a in $y = ax^2 + bx + c$ is 0? **If there is no squared term, $a = 0$.**

If the values of a, b, and c in $y = ax^2 + bx + c$ are all positive, what do you know about the graph? **The graph opens up because a is positive; and the vertex is to the left of the y-axis because $-\frac{b}{2a}$ is negative.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Reasoning

MP.2 Point out to students that an equation must be solved for y in terms of x before they determine whether the equation represents a quadratic function. For example $y + x^2 = x^2 + 4x + 1$ does not represent a quadratic function. Even though there is an x^2 term in the equation, when the equation is solved for y and simplified, there is no x^2 term.

Explain 1 Identifying Quadratic Functions in Standard Form

If a function is quadratic, it can be represented by an equation of the form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. This is called the **standard form of a quadratic equation**.

The axis of symmetry for a quadratic equation in standard form is given by the equation $x = -\frac{b}{2a}$. The vertex of a quadratic equation in standard form is given by the coordinates $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Example 1 **Determine whether the function represented by each equation is quadratic. If so, give the axis of symmetry and the coordinates of the vertex.**

(A) $y = -2x + 20$

$y = -2x + 20$ Compare to $y = ax^2 + bx + c$.

This is not a quadratic function because $a = 0$.

(B) $y + 3x^2 = -4$

Rewrite the function in the form $y = ax^2 + bx + c$.

$y =$ **$-3x^2 - 4$**

Compare to $y = ax^2 + bx + c$.

This **is** a quadratic function.

If $y + 3x^2 = -4$ is a quadratic function, give the axis of symmetry. **$x = 0$**

If $y + 3x^2 = -4$ is a quadratic function, give the coordinates of the vertex. **$(0, -4)$**

Reflect

3. Explain why the function represented by the equation $y = ax^2 + bx + c$ is quadratic only when $a \neq 0$.
If $a = 0$, there is no x-squared term and all quadratic equations include an x-squared term.

4. Why might it be easier to determine whether a function is quadratic when it is expressed in function notation?
All the terms are already on one side of the equal sign; they only need to be arranged in standard form.

5. How is the axis of symmetry related to standard form?
The axis of symmetry for a quadratic equation in standard form is given by the equation $x = -\frac{b}{2a}$, where a and b are constants in $y = ax^2 + bx + c$.

Your Turn

Determine whether the function represented by each equation is quadratic.

6. $y - 4x + x^2 = 0$
$y = -x^2 + 4x$
$y - 4x + x^2 = 0$ is a quadratic function.

7. $x + 2y = 14x + 6$
$y = \frac{13}{2}x + 3$
Compare $y = \frac{13}{2}x + 3$ to $y = ax^2 + bx + c$.
This is not a quadratic function because $a = 0$.

COLLABORATIVE LEARNING

Peer to Peer Activity

Have students work with a partner. Have each partner make up a quadratic function in vertex form. Then have students exchange papers and convert the vertex form to standard form, showing each step in the solution. Encourage students to repeat this several times, using different signs and reorganizing the equation to make it more difficult. For example, "Convert $y - 3 = (x - 5)^2$ to standard form." **$y = (x - 5)^2 + 3; y = x^2 - 10x + 25 + 3; y = x^2 - 10x + 28$**

Explain 2 Changing from Vertex Form to Standard Form

It is possible to write quadratic equations in various forms.

Example 2 **Rewrite a quadratic function from vertex form, $y = a(x - h)^2 + k$, to standard form, $y = ax^2 + bx + c$.**

Ⓐ $y = 4(x - 6)^2 + 3$

$y = 4(x^2 - 12x + 36) + 3$ Expand $(x - 6)^2$.

$y = 4x^2 - 48x + 144 + 3$ Multiply.

$y = 4x^2 - 48x + 147$ Simplify.

The standard form of $y = 4(x - 6)^2 + 3$ is $y = 4x^2 - 48x + 147$.

Ⓑ $y = -3(x + 2)^2 - 1$

$y = -3(\boxed{x^2 + 4x + 4}) - 1$ Expand $(x + 2)^2$.

$y = \boxed{-3x^2 - 12x - 12} - 1$ Multiply.

$y = \boxed{-3x^2 - 12x - 13}$ Simplify.

The standard form of $y = -3(x + 2)^2 - 1$ is $y = \boxed{-3x^2 - 12x - 13}$.

Reflect

8. If in $y = a(x - h)^2 + k$, $a = 1$, what is the simplified form of the standard form, $y = ax^2 + bx + c$?

$y = x^2 + bx + c$

Your Turn

Rewrite a quadratic function from vertex form, $y = a(x - h)^2 + k$, to standard form, $y = ax^2 + bx + c$.

9. $y = 2(x + 5)^2 + 3$

$y = 2(x^2 + 10x + 25) + 3$
$y = 2x^2 + 20x + 50 + 3$
$y = 2x^2 + 20x + 53$
The standard form of $y = 2(x + 5)^2 + 3$ is $y = 2x^2 + 20x + 53$.

10. $y = -3(x - 7)^2 + 2$

$y = -3(x^2 - 14x + 49) + 2$
$y = -3x^2 + 42x - 147 + 2$
$y = -3x^2 + 42x - 145$
The standard form of $y = -3(x - 7)^2 + 2$ is $y = -3x^2 + 42x - 145$.

DIFFERENTIATE INSTRUCTION

Technology

Have students set **Xmin** = 0 and **Ymin** = 0 when graphing the example on a graphing calculator. **Ymax** should be set to slightly above the maximum height. To generate a table from the function, set **TblStart** to 0.

EXPLAIN 2

Changing from Vertex Form to Standard Form

QUESTIONING STRATEGIES

When is the vertex form of a quadratic function useful? When is standard form useful? **Vertex form makes it easy to identify the vertex, (h, k), and the axis of symmetry, $x = h$. Standard form makes it easy to identify the y-intercept, c.**

How do you change the vertex form to standard form? **Use the FOIL method to expand and simplify the binomial square.**

What value is the same in vertex form as in standard form? Why? **The value of a. If you expand $a(x - h)^2 + k$, the coefficient of x^2 is a.**

MODELING

Have students consider what value is the same in vertex form as in standard form. Students comfortable with manipulating variables and constants might try to expand $a(x - h)^2 + k$ to determine the values of a, b, and c in terms of a, h, and k. They should find:

$a(x - h)^2 + k$

$a(x^2 - 2hx + h^2) + k$

$ax^2 - 2ahx + ah^2 + k$

So, $a = a$, $b = -2ah$, and $c = ah^2 + k$.

EXPLAIN 3

Writing a Quadratic Function Given a Table of Values

QUESTIONING STRATEGIES

What do you look for in a table of values in order to decide that it is quadratic? **A minimum or maximum value of y, indicating the vertex and axis of symmetry, and values of y that are equal for x-values that are the same distance from the vertex.**

AVOID COMMON ERRORS

Some students may overlook the negative sign in $-\frac{b}{2a}$ when calculating the axis of symmetry. Suggest that students double check that they included the negative sign before finding the vertex of the graph.

Explain 3 Writing a Quadratic Function Given a Table of Values

You can write a quadratic function from a table of values.

Example 3 **Use each table to write a quadratic function in vertex form, $y = a(x - h)^2 + k$. Then rewrite the function in standard form, $y = ax^2 + bx + c$.**

Ⓐ The minimum value of the function occurs at $x = -3$.

x	y
−6	9
−4	1
−3	0
−2	1
0	9

The vertex of the parabola is $(-3, 0)$.

Substitute the values for h and k into $y = a(x - h)^2 + k$.

$y = a(x - (-3))^2 + 0$, or $y = a(x + 3)^2$

Use any point from the table to find a.

$y = a(x + 3)^2$

$1 = a(-2 + 3)^2 = a$

The vertex form of the function is $y = 1(x - (-3))^2 + 0$ or $y = (x + 3)^2$.

Rewrite the function $y = (x + 3)^2$ in standard form, $y = ax^2 + bx + c$.

$y = (x + 3)^2 = x^2 + 6x + 9$

The standard form of the function is $y = x^2 + 6x + 9$.

Ⓑ The minimum value of the function occurs at $x = -2$.

x	y
0	13
−1	1
−2	−3
−3	1
−4	13

The vertex of the parabola is is $(-2, -3)$.

Substitute the values for h and k into $y = a(x - h)^2 + k$.

$y = \boxed{a(x + 2)^2 - 3}$

Use any point from the table to find a. $a = \boxed{4}$

The vertex form of the function is $y = \boxed{4(x + 2)^2 - 3}$.

Rewrite the resulting function in standard form, $y = ax^2 + bx + c$.

$y = \boxed{4x^2 + 16x + 13}$

Reflect

11. How many points are needed to find an equation of a quadratic function?
two points, the vertex and one other point

LANGUAGE SUPPORT EL

Communicate Math

Hand out cards to pairs of students with three numbers between −5 and 5 for a, h, and k; for example $a = 2$, $h = -4$, and $k = 1$. Have one student describe to the other how to write the function in vertex form, and then convert it to standard form. Then have them reverse roles and use the opposites of the numbers (in this example, $a = -2$, $h = 4$, and $k = -1$) to write a new function in vertex form, and convert that to standard form. Then have students describe how the graphs of these functions would be related to each other.

Your Turn

Use each table to write a quadratic function in vertex form, $y = a(x - h)^2 + k$. Then rewrite the function in standard form, $y = ax^2 + bx + c$.

12. The vertex of the parabola is (2, 5).

x	y
−1	59
1	11
2	5
3	11
5	59

$y = a(x - 2)^2 + 5$
$11 = a(1 - 2)^2 + 5$
$6 = a(-1)^2$
$6 = a$
vertex form $y = 6(x - 2)^2 + 5$.
$y = 6(x^2 - 4x + 4) + 5$
$y = 6x^2 - 24x + 24 + 5$
$y = 6x^2 - 24x + 29$
standard form $y = 6x^2 - 24x + 29$.

13. The vertex of the parabola is $(-2, -7)$.

x	y
0	−27
−1	−12
−2	−7
−3	−12
−4	−27

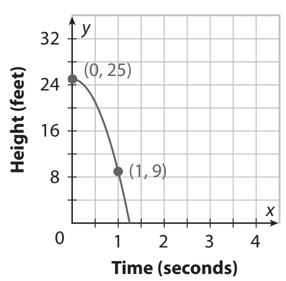
$y = a(x + 2)^2 - 7$
$-12 = a(-3 + 2)^2 - 7$
$-5 = a(-1)^2$
$-5 = a$
vertex form $y = -5(x + 2)^2 - 7$.
$y = -5(x^2 + 4x + 4) - 7$
$y = -5x^2 - 20x - 20 - 7$
$y = -5x^2 - 20x - 27$
standard form $y = -5x^2 - 20x - 27$.

Explain 4 Writing a Quadratic Function Given a Graph

The graph of a parabola can be used to determine the corresponding function.

Example 4 **Use each graph to find an equation for $f(t)$.**

Ⓐ A house painter standing on a ladder drops a paintbrush, which falls to the ground. The paintbrush's height above the ground (in feet) is given by a function of the form $f(t) = a(t - h)^2$ where t is the time (in seconds) after the paintbrush is dropped.



The vertex of the parabola is $(h, k) = (0, 25)$.

$f(t) = a(x - h)^2 + k$

$f(t) = a(t - 0)^2 + 25$

$f(t) = at^2 + 25$

Use the point (1, 9) to find a.

$f(t) = at^2 + 25$

$9 = a(1)^2 + 25$

$-16 = a$

The equation for the function is $f(t) = -16t^2 + 25$.

EXPLAIN 4

Writing a Quadratic Function Given a Graph

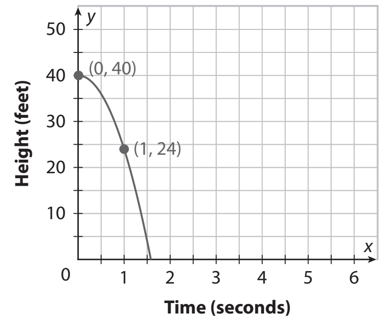
QUESTIONING STRATEGIES

? In order to find the equation of the graph of an object that is dropped and falls from a height over time, what values can you use? the y-coordinate of the vertex, and the coordinates of another point on the curve

VISUAL CUES

Remind students that, when graphing a curve of an object in free fall, the graph represents the distance the object is above the surface and not the path of the object.

(B) A rock is knocked off a cliff into the water far below. The falling rock's height above the water (in feet) is given by a function of the form $f(t) = a(t - h)^2 + k$ where t is the time (in seconds) after the rock begins to fall.



The vertex of the parabola is $(h, k) = \boxed{(0, 40)}$.

$f(t) = a(t - h)^2 + k$

$f(t) = a\left(t - \boxed{0}\right)^2 + \boxed{40}$.

$f(t) = \boxed{at^2 + 40}$

Use the point $\underline{(1, 24)}$ to find a.

$f(t) = at^2 + \boxed{40}$

$\boxed{24} = a\boxed{1}^2 + \boxed{40}$

$a = \boxed{-16}$

The equation for the function is $f(t) = \boxed{-16t^2 + 40}$.

Reflect

14. Identify the domain and explain why it makes sense for this problem.

The domain is $t \geq 0$.

Time cannot be negative.

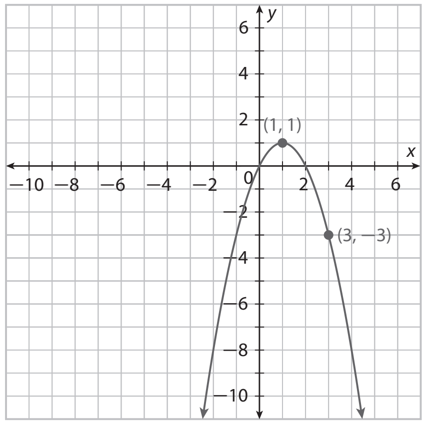
15. Identify the range and explain why it makes sense for this problem.

The range is $f(t) \geq 0$.

Height cannot be negative.

Your Turn

16. The graph of a function in the form $f(x) = a(x - h)^2 + k$, is shown. Use the graph to find an equation for $f(x)$.



The vertex of the parabola is $(h, k) = (1, 1)$.

$$f(x) = a(x - 1)^2 + 1$$

From the graph $f(3) = -3$ Substitute 3 for x and -3 for $f(x)$ and solve for a.

$$-3 = a(3 - 1)^2 + 1$$

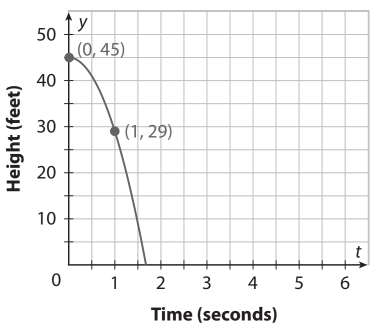
$$-4 = a(2)^2$$

$$-4 = 4a$$

$$-1 = a$$

The equation for the function is $f(x) = -(x - 1)^2 + 1$.

17. A roofer accidentally drops a nail, which falls to the ground. The nail's height above the ground (in feet) is given by a function of the form $f(t) = a(t - h)^2 + k$, where t is the time (in seconds) after the nail drops. Use the graph to find an equation for $f(t)$.



The vertex of the parabola is $(h, k) = (0, 45)$.

$f(t) = a(t - 0)^2 + 45$, **or** $f(t) = at^2 + 45$

From the graph $f(1) = 29$. Substitute 1 for t and 29 for $f(t)$ and solve for a.

$$29 = a(1)^2 + 45$$

$$-16 = a$$

The equation for the function is $f(t) = -16t^2 + 45$.

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Have two students hold the ends of a jump rope at the same height from the ground. Use a measuring tape to place the students 4 feet, 6 feet, and 8 feet apart. Have the students verify whether the lowest point of the jump rope is always halfway between the students holding the rope. Students can graph the height of the rope based on the distance from either end and determine appropriate quadratic functions to describe it. Mention that the curve is called a *catenary*, but that because the shape of a catenary is similar to the shape of a parabola, a quadratic model is appropriate.

SUMMARIZE THE LESSON

How can you change the vertex form of a quadratic function to standard form? **You can rewrite the quadratic expression in the vertex form by multiplying and then combining like terms so that the function rule is written in descending order of the exponents.**

Elaborate

18. Describe the graph of a quadratic function.
parabola

19. What is the standard form of the quadratic function?
$y = ax^2 + bx + c$

20. Can any quadratic function in vertex form be written in standard form?
yes

21. How many points are needed to write a quadratic function in vertex form, given the table of values?
two points; vertex and another point

22. If a graph of the quadratic function is given, how do you find the vertex?
Look for the minimum or maximum value on the graph.

23. **Essential Question Check-In** What can you do to change the vertex form of a quadratic function to standard form?
Sample answer: You can rewrite the quadratic expression in the vertex form by multiplying and then combine like terms so that the function rule is written in descending order of the exponents.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Determine whether each function is a quadratic function by graphing.

1. $f(x) = 0.01 - 0.2x + x^2$
The graph is a parabola. It is quadratic.

2. $f(x) = \frac{1}{2}x - 4$
The graph is not a parabola. It is not quadratic.

3. $f(x) = -4x^2 - 2$
The graph is a parabola. It is quadratic.

4. $f(x) = 2^{x-3}$
The graph is not a parabola. It is not quadratic.

Determine whether the function represented by each equation is quadratic.

5. $y = -3x + 15$
No, $a = 0$

6. $y - 6 = 2x^2$
$y = 2x^2 + 6$
Yes, $a \neq 0$, b, and c are real numbers.

7. $3 + y + 5x^2 = 6x$
$y = -5x^2 + 6x - 3$
Yes, $a \neq 0$, b, and c are real numbers.

8. $y + 6x = 14$
$y = -6x + 14$
No, $a = 0$

9. Which of the following functions is a quadratic function? Select all that apply.

a. $2x = y + 3$
b. $2x^2 + y = 3x - 1$
c. $5 = -6x + y$
d. $6x^2 + y = 0$
e. $y - x = 4$

10. For $f(x) = x^2 + 8x - 14$, give the axis of symmetry and the coordinates of the vertex.

$-\frac{b}{2a} = -\frac{8}{2(1)} = -4$

$f(-4) = -30$

The vertex is at $(-4, -30)$.

The axis of symmetry is $x = -4$.