LESSON 21.1

Solving Equations by Factoring $x^2 + bx + c$

Common Core Math Standards

The student is expected to:

CA CC **A-SSE.2**

Use the structure of an expression to identify ways to rewrite it. Also A-SSE.2, A-REI.4b

Mathematical Practices

CA CC **MP.7 Using Structure**

Language Objective

Explain to a partner how to solve $x^2 - 6x - 16 = 0$.

ENGAGE

Essential Question: How can you use factoring to solve quadratic equations in standard form for which $a = 1$?

Factor the quadratic equation. Set each linear factor equal to 0. Solve each linear equation. The solutions are the solutions of the original quadratic equation.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss how green roofs can help control storm-water runoff and reduce stress on sewer systems. Then preview the Lesson Performance Task.

Name ______________ Class ______________ Date ______________

21.1 Solving Equations by Factoring $x^2 + bx + c$

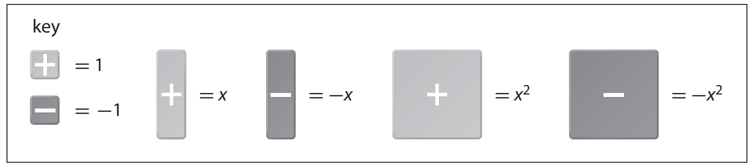
Resource Locker

Essential Question: How can you use factoring to solve quadratic equations in standard form for which $a = 1$?

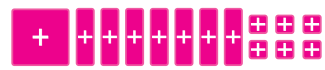
Explore 1 Using Algebra Tiles to Factor $x^2 + bx + c$

In this lesson, multiplying binomials using the FOIL process will be reversed and trinomials will be factored into two binomials. To learn how to factor, let's start with the expression $x^2 + 7x + 6$.

(A) Identify and draw the tiles needed to model the expression $x^2 + 7x + 6$.

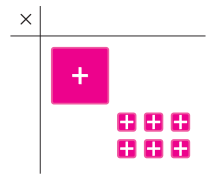


The tiles needed to model the expression $x^2 + 7x + 6$ are:

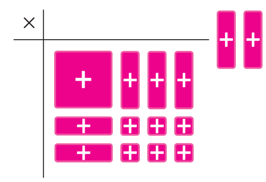


__1__ x^2-tiles(s), __7__ x-tile(s), and __6__ unit tile(s).

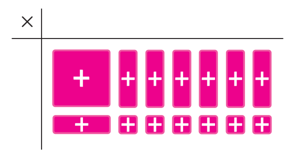
(B) Arrange and draw the algebra tiles on the grid. Place the __1__ x^2-tile(s) in the upper left corner and arrange the __6__ unit tiles in two rows and three columns in the lower right corner.

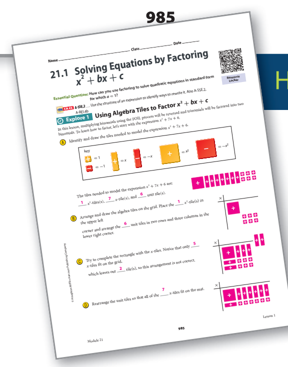


(C) Try to complete the rectangle with the x-tiles. Notice that only __5__ x-tiles fit on the grid, which leaves out __2__ tile(s), so this arrangement is not correct.



(D) Rearrange the unit tiles so that all of the __7__ x-tiles fit on the mat.



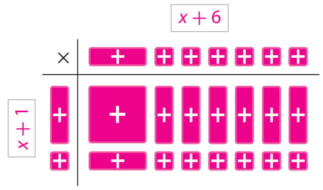


HARDCOVER PAGES 771–780

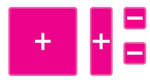
Turn to these pages to find this lesson in the hardcover student edition.

Ⓔ Complete the multiplication grid by placing the factor tiles on the sides. Then write the factors modeled in this product.

$x^2 + 7x + 6 = (x + \boxed{1})(x + \boxed{6})$



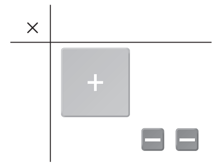
Ⓕ Now let's look at how to factor a quadratic expression with a negative constant term. Use algebra tiles to factor $x^2 + x - 2$. Identify the tiles needed to model the expression.



1 positive x^2-tile(s), 1 positive x-tile(s), and 2 negative unit tile(s)

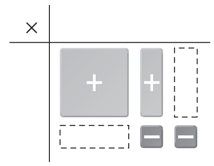
Ⓖ Arrange the algebra tiles on the grid. Place the 1 positive x^2-tile(s) in the upper left

corner and arrange the 2 negative unit tiles in the lower right corner.

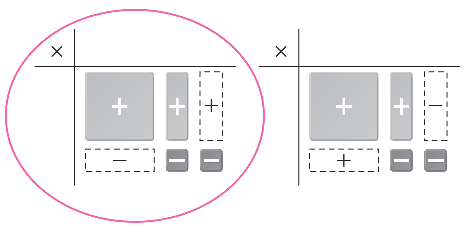


Ⓗ Try to fill in the empty spaces on the grid with x-tiles. There is/are 1 positive x-tile(s) to

place on the grid, so there will be 2 empty places for x-tiles.

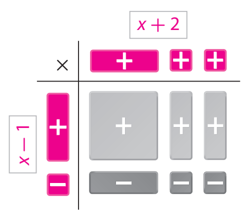


Ⓘ Complete the rectangle on the mat by using *zero pairs*. Add 1 positive x-tile(s) and 1 negative x-tile(s) to the grid in such a way that the factors work with all the tiles on the mat. Circle the mat showing the correct position of zero pairs.



Ⓙ Complete the multiplication grid by placing the factor tiles on the sides. Then write the factors modeled in this product.

$x^2 + x - 2 = (x + \boxed{2})(x - \boxed{1})$



EXPLORE 1

Using Algebra Tiles to Factor $x^2 + bx + c$

INTEGRATE TECHNOLOGY

Students have the option of completing the algebra tiles activity either in the book or online.

QUESTIONING STRATEGIES

When using algebra tiles to factor a polynomial, when do you need to add a zero pair of x-tiles? **You need to add a zero pair when there are empty spaces in the grid but there is no way to rearrange the tiles that are already there to fill those spaces. By adding one positive x-tile and one negative x-tile, you fill in the grid without changing the value of the original polynomial.**

PROFESSIONAL DEVELOPMENT

Learning Progressions

In this lesson, students learn to reverse the process of multiplying two binomials with FOIL to find two binomials whose product is a given trinomial. Some key understandings for students are as follows:

- To factor a trinomial $x^2 + bx + c$, list factor pairs of c, then use the factor pair whose sum is equal to b to factor the trinomial.
- To solve an equation of the form $x^2 + bx + c = 0$, factor the trinomial, then set each factor equal to 0 and solve for x.

EXPLORE 2

Factoring $x^2 + bx + c$

QUESTIONING STRATEGIES

How are the terms in a trinomial of the form $x^2 + bx + c$ related to the four products in the FOIL method (First, Outer, Inner, Last)? **The x^2-term is the product of the two First terms in the binomial. The term *bx* is the sum of the products of the Outer terms and the Inner terms. The constant *c* is the product of the two Last terms in the two binomials.**

VISUAL CUES

When factoring a trinomial of the form $x^2 + bx + c$, suggest that students start by writing down what the product and sum of the two constants in the binomials should be. They can write P for product and S for sum next to the trinomial as a reminder to complete this step. Remind students to include the sign for each product or sum as they write it.

Reflect

1. Are there any other ways to factor the polynomial $x^2 + 7x + 6$ besides $(x + 6)(x + 1)$? Explain.
   No, 2-by-3 was also checked.

2. **Discussion** If c is positive in $x^2 + bx + c$, what sign can the constant terms of the factors have? What about when c is negative?
   If *c* is positive, then both factors are positive or both factors are negative.
   If *c* is negative, the factors have opposite signs.

Explore 2 Factoring $x^2 + bx + c$

To factor $x^2 + bx + c$, you need to find two factors of c whose sum is b.

Factoring $x^2 + bx + c$	
WORDS	**EXAMPLE**
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b. If no such Integers exist, the trinomial is not factorable.	To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9. Factors of 18 — Sum 1 and 18 — 19 — x 2 and 9 — 11 — x 3 and 6 — 9 — ✓ $x^2 + 9x + 18 = (x + 3)(x + 6)$

If c is positive, the constant terms of the factors have the same sign.

If c is negative, then one constant term of the factors is positive and one is negative.

Ⓐ First, look at $x^2 + 11x + 30$. Find the values of b and c. $b =$ **11** $c =$ **30**

Ⓑ c is positive / negative. The sign of the factors will be the same / different.

Ⓒ List the factor pairs of c, 30, and find the sum of each pair.

Factors of 30	Sum of Factors
1 and **30**	1 + **30** = **31**
2 and **15**	2 + **15** = **17**
3 and **10**	3 + **10** = **13**
5 and **6**	5 + **6** = **11**

Ⓓ The factor pair whose sum equals b is **5 and 6**.

Use this factor pair to factor the polynomial. $x^2 + 11x + 30 = (x + \mathbf{5})(x + \mathbf{6})$

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have students work in pairs. Each student should write two binomial factors of the form $(x + a)$ or $(x - a)$ and multiply them together to form a trinomial. Then have partners factor each other's trinomials. Have students check each other's work and discuss what makes some trinomials easier or harder to factor than others.

(E) Now, look at $x^2 + 13x - 30$. Find the values of b and c.

$b =$ **13** $\quad c =$ **−30**

(F) c is positive / negative. The sign of the factors will be the same / different.

(G) List the factor pairs of c, −30, and find the sum of each pair.

Factors of −30	Sum of Factors
1 and **−30**	1 + **−30** = **−29**
2 and **−15**	2 + **−15** = **−13**
3 and **−10**	3 + **−10** = **−7**
5 and **−6**	5 + **−6** = **−1**
−1 and **30**	−1 + **30** = **29**
−2 and **15**	−2 + **15** = **13**
−3 and **10**	−3 + **10** = **7**
−5 and **6**	−5 + **6** = **1**

(H) The factor pair whose sum equals b is **15 and −2**.

Use this factor pair to factor the polynomial.

$x^2 + 13x - 30 = (x + \mathbf{15})(x - \mathbf{2})$

Reflect

3. **Discussion** When factoring a trinomial of the form $x^2 + bx + c$, where c is negative, one binomial factor contains a positive factor of c and one contains a negative factor of c. How do you know which factor of c should be positive and which should be negative?

If b is positive, the factor of c with the greater absolute value must be positive. If b is negative, the factor of c with the greater absolute value is negative.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Discuss with students how they can use the signs of b and c in $x^2 + bx + c$ to determine whether to use positive or negative factors of c to factor the trinomial. Work together to complete a table like the following:

	Sign of Constant Terms in Binomial Factors of $x^2 + bx + c$
$c > 0$, $b > 0$	both constants positive
$c > 0$, $b < 0$	both constants negative
$c < 0$	one constant positive, one constant negative

DIFFERENTIATE INSTRUCTION

Critical Thinking

Show students how to create quadratic equations with given solutions by working backward. For example, if the solutions are −2 and 5, then $x = -2$ or $x = 5$. The factors can be written as $(x + 2)(x - 5)$, which is equivalent to $x^2 - 3x - 10$. Therefore, the quadratic equation is $x^2 - 3x - 10 = 0$. Explain that an infinite number of equivalent equations can be generated by performing the same operation on both sides of this equation.

EXPLAIN 1

Solving Equations of the Form $x^2 + bx + c = 0$ by Factoring

AVOID COMMON ERRORS

When solving quadratic equations by factoring, students sometimes make the mistake of naming the constants in the binomial factors as the solutions. Remind students that they need to find the value of x that would make each factor equal to 0. For example, the solutions of $(x - 6)(x - 4) = 0$ are 6 and 4, not -6 and -4.

Explain 1 Solving Equations of the Form $x^2 + bx + c = 0$ by Factoring

As you have learned, the Zero Product Property can be used to solve quadratic equations in factored form.

Example 1 **Solve each equation by factoring. Check your answer by graphing.**

(A) $x^2 - 8x = -12$

First, write the equation in the form $x^2 - bx + c = 0$.

$x^2 - 8x = -12$ Original equation

$x^2 - 8x + 12 = 0$ Add 12 to both sides.

The expression $x^2 - 8x + 12$ is in the form $ax^2 + bx + c$, with $b < 0$ and $c > 0$, so the factors will have the same sign and they both will be negative.

Factors of 12	Sum of Factors
-1 and -12	$-1 + (-12) = -13$
-2 and -6	$-2 + (-6) = -8$
-3 and -4	$-3 + (-4) = -7$

The factor pair whose sum equals -8 is -2 and -6. Factor the equation, and use the Zero Product Property.

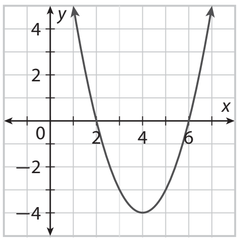
$$x^2 - 8x + 12 = 0$$

$$(x - 2)(x - 6) = 0$$

$$x - 2 = 0 \quad \text{or} \quad x - 6 = 0$$

$$x = 2 \qquad\qquad x = 6$$

The zeros of the equation are 2 and 6. Check this by graphing the related function, $f(x) = x^2 - 8x + 12$.



The x-intercepts of the graph are 2 and 6, which are the same as the zeros of the equation. The solutions of the equation are 2 and 6.

LANGUAGE SUPPORT EL

Auditory Cues

Have students explain how to factor $x^2 + bx + c$ when c is positive. Write out and model sentence frames for students to structure their answers. For example:

First find two numbers whose _______ is c and whose ______ is b. If c is positive, then both numbers are ______ or both are ______. If b is positive, then I choose the _______ factors of c. If b is negative, then I choose the _______ factors of c.

(B) $x^2 - 2x = 15$

First, rewrite the expression in the form $x^2 + bx + c = 0$.

$x^2 - 2x = 15$ Original equation

$x^2 - 2x - \boxed{15} = 0$ Subtract 15 from both sides.

To find the zeros of the equation, start by factoring. List the factor pairs of c and find the sum of each pair. Since $c < 0$, the factors will have opposite signs. Since $c < 0$ and $b < 0$, the factor with the greater absolute value will be negative.

Factors of -15	Sum of Factors
1 and $\boxed{-15}$	$1 + \boxed{-15} = \boxed{-14}$
3 and $\boxed{-5}$	$3 + \boxed{-5} = \boxed{-2}$
-1 and $\boxed{15}$	$-1 + \boxed{15} = \boxed{14}$
-3 and $\boxed{5}$	$-3 + \boxed{5} = \boxed{2}$

The factor pair whose sum equals -2 is 3 and -5. Factor the equation, and use the Zero Product Property.

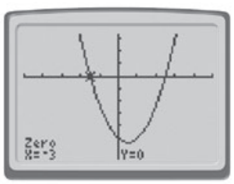
$$x^2 - 2x - 15 = 0$$

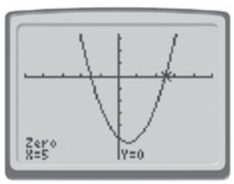
$$\left(x + \boxed{3}\right)\left(x - \boxed{5}\right) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = \boxed{-3} \qquad x = \boxed{5}$$

The zeros of the equation are $\boxed{-3}$ and $\boxed{5}$. Check this by graphing the related function, $f(x) = x^2 - 2x = 15$.





Your Turn

Solve each equation.

4. $x^2 + 15x = -54$

$x^2 + 15x + 54 = 0$
$(x + 6)(x + 9) = 0$
$x + 6 = 0$ or $x + 9 = 0$
$x = -6$ $x = -9$

5. $x^2 - 13x = -12$

$x^2 - 13x + 12 = 0$
$(x - 12)(x - 1) = 0$
$x - 12 = 0$ or $x - 1 = 0$
$x = 12$ $x = 1$

6. $x^2 - x = 56$

$x^2 - x - 56 = 0$
$(x + 7)(x - 8) = 0$
$x + 7 = 0$ or $x - 8 = 0$
$x = -7$ $x = 8$

QUESTIONING STRATEGIES

After factoring a polynomial to solve an equation, why do you set each factor equal to zero? **You set each factor equal to zero because, by the Zero Product Property, at least one of the factors must be zero if the product is equal to zero.**

EXPLAIN 2

Solving Equation Models of the Form $x^2 + bx + c = 0$ by Factoring

QUESTIONING STRATEGIES

? A frame has dimensions $(x + 4)$ inches and $(x + 7)$ inches and a total area of 108 square inches. What polynomial do you need to factor in order to find the dimensions of the frame? Explain. **$x^2 + 11x - 80$; you need to solve the equation $(x + 4)(x + 7) = 108$. To solve, rewrite it as $x^2 + 11x + 28 = 108$.**

AVOID COMMON ERRORS

Students may forget to interpret the solutions using the context of the problem. Remind students that values that do not make sense in the context of the problem should not be listed as solutions.

ELABORATE

QUESTIONING STRATEGIES

? Alonso says the product of two binomials is always a trinomial. Max says that it is possible to multiply two binomials and get a product that is still a binomial. Who is correct? **Max is correct. If you multiply two binomials with the same values but the opposite sign, the result is a binomial, for example $(x + 4)(x - 4) = x^2 - 16$.**

SUMMARIZE THE LESSON

Have students copy the graphic organizer and then complete it by writing a step used to solve a quadratic equation in each box.

Solve Quadratic Equations by Factoring

1. Factor.
2. **Set each factor equal to zero.**
3. **Solve each new equation.**

Explain 2 Solving Equation Models of the Form $x^2 + bx + c = 0$ by Factoring

Some real-world problems can be solved by factoring a quadratic equation.

Example 2 **Solve each model by factoring.**

Architecture A rectangular porch has dimensions of $(x + 12)$ and $(x + 5)$ feet. If the area of the porch floor is 120 square feet, what are its length and width?

Write an equation for the problem. Substitute 120 for A for the area of the porch.

$$(x + 12)(x + 5) = A$$
$$x^2 + 17x + 60 = A$$
$$x^2 + 17 + 60 = 120$$
$$x^2 + 17x - 60 = 0$$

The factors are of -60 that have a sum of 17 are 20 and -3. Use Zero Product Property to find x.

$$(x + 20)(x - 3) = 0$$
$$x + 20 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -20 \qquad x = 3$$

Since the area cannot be negative, $x = 3$ feet.

Therefore, the dimensions of the porch are $3 + 12 = 15$ feet long and $3 + 5 = 8$ feet wide.

Elaborate

7. How are the solutions of a quadratic equation related to the zeros of the related function?
By the Zero Product Property, the solutions found from the factored form of a quadratic equation are the zeros of the related function.

8. **Essential Question Check-In** How can you solve a quadratic equation by factoring?
Write the equation in standard form and find the factors of c that add to b. Use those factors to write the equation in factored form. Use the Zero Product Property to solve each linear equation.