LESSON 20.1

Connecting Intercepts and Zeros

Common Core Math Standards

The student is expected to:

CA CC F-IF.7a

Graph linear and quadratic functions and show intercepts, maxima, and minima. Also A-REI.4, A-APR.3, A-REI.11, F-LE.6

Mathematical Practices

CA CC MP.5 Using Tools

Language Objective

Given a quadratic function modeling a real-world situation, explain to a partner what the zeros of the function represent.

ENGAGE

Essential Question: How can you use the graph of a quadratic function to solve its related quadratic equation?

You can write the equation with one side equal to 0, graph the related function, and find the zeros of the function.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss what type of path is made by a diver diving into the water when her initial height is the height of the diving platform. Then preview the Lesson Performance Task.

Name ______________________ Class ______________ Date ________

20.1 Connecting Intercepts and Zeros

Resource Locker

Essential Question: How can you use the graph of a quadratic function to solve its related quadratic equation?

Explore Graphing Quadratic Functions in Standard Form

A parabola can be graphed using its vertex and axis of symmetry. Use these characteristics, the y-intercept, and symmetry to graph a quadratic function.

Graph $y = x^2 - 4x - 5$ by completing the steps.

(A) Find the axis of symmetry.

$x = -\frac{b}{2a}$

$= -\frac{\boxed{-4}}{2 \cdot \boxed{1}}$

$= \boxed{2}$

The axis of symmetry is $x = \boxed{2}$.

(B) Find the vertex.

$y = x^2 - 4x - 5$

$= \boxed{2}^2 - 4 \cdot \boxed{2} - 5$

$= \boxed{4} - \boxed{8} - 5$

$= \boxed{-9}$

The vertex is $(\boxed{2}, \boxed{-9})$.

(C) Find the y-intercept.

$y = x^2 - 4x - 5$

$y = \boxed{0}^2 - 4\boxed{0} + (\boxed{-5})$

The y-intercept is $\boxed{-5}$; the graph passes through $(0, \boxed{-5})$.

(D) Find two more points on the same side of the axis of symmetry as the y-intercept.

a. Find y when $x = 1$.

$y = x^2 - 4x - 5$

$= \boxed{1}^2 - 4 \cdot \boxed{1} - 5$

$= \boxed{1} - \boxed{4} - 5$

$= \boxed{-8}$

The first point is $(\boxed{1}, \boxed{-8})$.

b. Find y when $x = -1$.

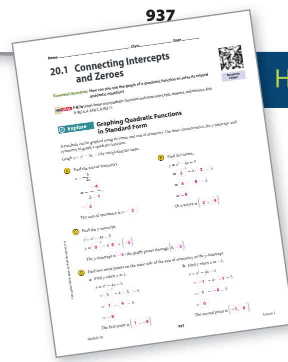
$y = x^2 - 4x - 5$

$= \boxed{-1}^2 - 4 \cdot (\boxed{-1}) - 5$

$= \boxed{1} - (\boxed{-4}) - 5$

$= \boxed{0}$

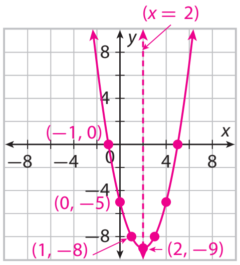
The second point is $(\boxed{-1}, \boxed{0})$.



HARDCOVER PAGES 735–744

Turn to these pages to find this lesson in the hardcover student edition.

Ⓔ Graph the axis of symmetry, the vertex, the y-intercept, and the two extra points on the same coordinate plane. Then reflect the graphed points over the axis of symmetry to create three more points, and sketch the graph.



Reflect

1. **Discussion** Why is it important to find additional points before graphing a quadratic function?
Additional points provide more information about the shape of the parabola, and the sketch of the quadratic function will be more accurate.

Explain 1 Using Zeros to Solve Quadratic Equations Graphically

A **zero of a function** is an x-value that makes the value of the function 0. The zeros of a function are the x-intercepts of the graph of the function. A quadratic function may have one, two, or no zeros.

Quadratic equations can be solved by graphing the related function of the equation. To write the related function, rewrite the quadratic equation so that it equals zero on one side. Replace the zero with y.

Graph the related function. Find the x-intercepts of the graph, which are the zeros of the function. The zeros of the function are the solutions to the original equation.

Example 1 **Solve by graphing the related function.**

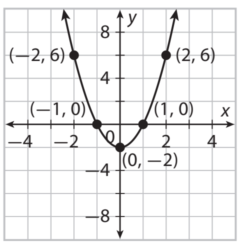
Ⓐ $2x^2 - 5 = -3$

a. Write the related function. Add 3 to both sides to get $2x^2 - 2 = 0$. The related function is $y = 2x^2 - 2$.

b. Make a table of values for the related function.

x	−2	−1	0	1	2
y	6	0	−2	0	6

c. Graph the points represented by the table and connect the points.



d. The zeros of the function are −1 and 1, so the solutions of the equation $2x^2 - 5 = -3$ are $x = -1$ and $x = 1$.

PROFESSIONAL DEVELOPMENT

Math Background

Quadratic equations are often used to model the motion of falling objects. The general formula for this motion is $h = -16t^2 + v_0t + h_0$, where h represents the height of the object in feet, t is the number of seconds the object has been falling, v_0 is the initial vertical velocity of the object in feet per second, and h_0 is the initial height of the object in feet. The coefficient −16 is equal to half of the constant acceleration due to gravity, $-32\ \text{ft/s}^2$. Students can compare the quadratic equations given for falling objects in this lesson to the general formula to determine the values of v_0 and h_0 in each case.

EXPLORE

Graphing Quadratic Functions in Standard Form

QUESTIONING STRATEGIES

? How many points on the graph of a quadratic function are on the axis of symmetry? Explain. **One; the only point on the graph of a quadratic function that is on the axis of symmetry is the vertex of the function.**

? Why is it helpful to find the axis of symmetry when graphing a quadratic function? **After you find the axis of symmetry and a few points on one side of it, you can use symmetry to quickly and easily find an equal number of points on the other side.**

EXPLAIN 1

Using Zeros to Solve Quadratic Equations Graphically

AVOID COMMON ERRORS

Students may think that the zeros of a quadratic function can be found by substituting 0 for x in the function. Make sure students understand that the zeros of a function are the values of x when $y = 0$, not the values of y when $x = 0$.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 For cases in which a quadratic function has two zeros, discuss with students how to use the zeros to determine the function's axis of symmetry. Students should realize that the axis of symmetry will be the x-value of the point that is midway between the two zeros.

QUESTIONING STRATEGIES

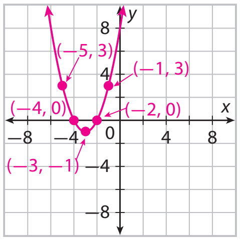
When a quadratic function has two zeros that are opposites, what must be true about the function? **The axis of symmetry must be $x = 0$, and the vertex must not be $(0, 0)$.**

Ⓑ $6x + 8 = -x^2$

a. Write the related function. Add x^2 to both sides to get $\boxed{x^2} + 6x + 8 = \boxed{0}$. The related function is $\boxed{y} = \boxed{x^2} + 6x + 8$.

b. Make a table of values for the related function.

x	−5	−4	−3	−2	−1
y	3	0	−1	0	3



c. Graph the points represented by the table and connect the points.

d. The zeros of the function are $\boxed{-4}$ and $\boxed{-2}$, so the solutions of the equation

$6x + 8 = -x^2$ are $x = \boxed{-4}$ and $x = \boxed{-2}$.

Reflect

2. How would the graph of a quadratic equation look if the equation has one zero?
If the quadratic equation has one zero, the graph will intersect the *x*-axis at its vertex.

Your Turn

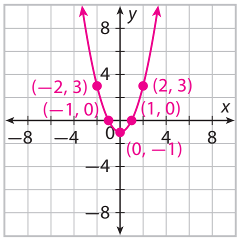
3. $x^2 - 4 = -3$

$x^2 - 4 = -3$

$x^2 - 1 = 0$

The related function is $y = x^2 - 1$.

Graph:



The zeros of the function are 1 and −1, so the solutions of the equation are $x = -1$ and $x = 1$.

COLLABORATIVE LEARNING

Peer-to-Peer Activity

Have students work in pairs. Have each student find the solution to the same quadratic equation by graphing. One student in each pair then solves the equation by finding the zeros of the function, and the other student uses points of intersection to find the solution. Students compare their answers; while the graphs may be different, the solution should be the same regardless of the method used.

Explain 2 Using Points of Intersection to Solve Quadratic Equations Graphically

You can solve a quadratic equation by rewriting the equation in the form $ax^2 + bx = c$ or $a(x - h)^2 = k$ and then using the expressions on each side of the equal sign to define a function.

Graph both functions and find the points of intersection. The solutions are the x-coordinates of the points of intersection on the graph. As with using zeros, there may be two, one, or no points of intersection.

Example 2 **Solve each equation by finding points of intersection of two related functions.**

(A) $2(x - 4)^2 - 2 = 0$

$2(x - 4)^2 = 2$ Write as $a(x - h)^2 = k$.

a. Let $f(x) = 2(x - 4)^2$. Let $g(x) = 2$.

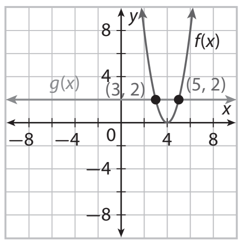
b. Graph $f(x)$ and $g(x)$ on the same graph.

c. Determine the points at which the graphs of $f(x)$ and $g(x)$ intersect.

The graphs intersect at two locations: $(3, 2)$ and $(5, 2)$.

This means $f(x) = g(x)$ when $x = 3$ and $x = 5$.

So the solutions of $2(x - 4)^2 - 2 = 0$ are $x = 3$ and $x = 5$.



(B) $3(x - 5)^2 - 12 = 0$

$3(x - 5)^2 = \boxed{12}$

a. Let $f(x) = \boxed{3}(x - 5)^2$. Let $g(x) = \boxed{12}$.

b. Graph $f(x)$ and $g(x)$ on the same graph.

c. Determine the points at which the graphs of $f(x)$ and $g(x)$ intersect.

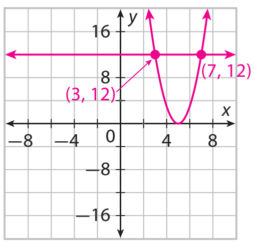
The graphs intersect at two locations:

$(\boxed{3}, \boxed{12})$ and $(\boxed{7}, \boxed{12})$.

This means $f(x) = g(x)$ when $x = \boxed{3}$ and $x = \boxed{7}$.

Therefore, the solutions of the equation $f(x) = g(x)$ are $\boxed{3}$ and $\boxed{7}$.

So the solutions of $3(x - 5)^2 - 12 = 0$ are $x = \boxed{3}$ and $x = \boxed{7}$.



DIFFERENTIATE INSTRUCTION

Communicating Math

Review the parameters of a parabola that students can use when graphing a function. Students should understand that being able to identify the vertex of a parabola will make it easier to graph the function, but they may have different ideas about how to find enough other points in order to make an accurate graph. Discuss how students can be sure that they have plotted enough points to sketch the function on a coordinate grid.

EXPLAIN 2

Using Points of Intersection to Solve Quadratic Equations Graphically

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Remind students that quadratic equations in the form $a(x - h)^2 + k$ will have the vertex at (h, k). After identifying the vertex, students can use substitution to find enough points to graph the quadratic function.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 When using points of intersection to find solutions to quadratic equations, make sure that students understand that only the x-values of the points of intersection are solutions to the equation. The y-values come from the function that was created to find the solutions; they are not part of the solution.

QUESTIONING STRATEGIES

How can you use graphing to determine that a quadratic equation has no solutions? **After rewriting the equation in the form $ax^2 + bx = c$, if the graphs of the functions $f(x) = ax^2 + bx$ and $g(x) = c$ do not intersect, then the quadratic function has no solutions.**

EXPLAIN 3

Modeling a Real-World Problem

INTEGRATE TECHNOLOGY

Before students are familiar with using the quadratic formula, graphing calculators offer the most accessible way to find the solutions to quadratic equations that do not have simple whole-number solutions.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 The functions used to determine the height of a thrown object can be difficult to understand. Discuss different ways to rewrite the functions used to determine height. Students may wish to rewrite a function like $h(t) = -16t^2 + 100$ as $h(t) = 100 - 16t^2$ to make it more evident that the object loses height for each second of time.

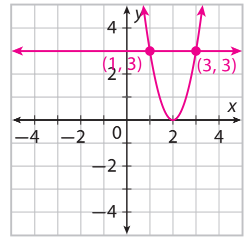
Reflect

4. In Part B above, why is the x-coordinates the answer to the equation and not the y-coordinates?

The x-coordinates are the solution because the x-values are the unknown amount being solved for in the original equation. The y-values are used to create a related function to find the x values, but since they are not part of the original equation, the y values are not part of the solution.

Your Turn

5. Solve $3(x - 2)^2 - 3 = 0$ by finding the points of intersection of the two related functions.



$\mathbf{3(x - 2)^2 - 3 = 0}$

$\mathbf{3(x - 2)^2 = 3}$

Let $f(x) = 3(x - 2)^2$ and let $g(x) = 3$.

The graphs intersect at two locations: $(1, 3)$ and $(3, 3)$.

This means $f(x) = g(x)$ when $x = 1$ and $x = 3$.

So the solutions of $3(x - 2)^2 - 3 = 0$ are 1 and 3.

Explain 3 Modeling a Real-World Problem

Many real-world problems can be modeled by quadratic functions.

Example 3 **Create a quadratic function for each problem and then solve it by using a graphing calculator.**

Nature A squirrel is in a tree holding a chestnut at a height of 46 feet above the ground. It drops the chestnut, which lands on top of a bush that is 36 feet below the squirrel. The function $h(t) = -16t^2 + 46$ gives the height in feet of the chestnut as it falls, where t represents time. When will the chestnut reach the top of the bush?

Analyze Information

Identify the important information.

- The chestnut is **46** feet above the ground, and the top of the bush is **36** feet below the chestnut.
- The chestnut's height as a function of time can be represented by $h(t) = \boxed{-16}\,t^2 + \boxed{46}$, where $(h)t$ is the height of the chestnut in feet as it is falling.

Formulate a Plan

Create a related quadratic equation to find the height of the chestnut in relation to time. Use $h(t) = -16t^2 + 46$ and insert the known value for h.

LANGUAGE SUPPORT EL

Connect Vocabulary

Students may expect that problems involving points of intersection will always have solutions. Relate the word *intersection* to its uses outside the math classroom. Just as two streets in a town may or may not intersect, the graphs of two functions may or may not intersect. When dealing with the intersection of a quadratic function and a linear function, remind students that there may be 0, 1, or 2 points of intersection.

Solve

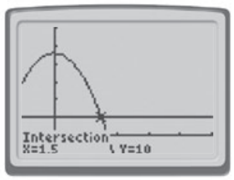
Write the equation that needs to be solved. Since the top of the bush is 36 feet below the squirrel, it is 10 feet above the ground.

$-16t^2 + 46 = 10$

Separate the function into $y = f(t)$ and $y = g(t)$. $f(t) =$ **−16** t **2** $+$ **46** and $g(t) =$ **10**.

To graph each function on a graphing calculator, rewrite them in terms of x and y.

$y =$ **−16** x **2** $+$ **46** and $y =$ **10**



Graph both functions. Use the intersect feature to find the amount of time it takes for the chestnut to hit the top of the bush.

The chestnut will reach the top of the bush in **1.5** seconds.

Justify and Evaluate

$-16\left(\boxed{1.5}\right)^2 + 46 = 10$

$\boxed{-36} + 46 = 10$

$\boxed{10} = \boxed{10}$

When t is replaced by **1.5** in the original equation, $-16t^2 + 46 = 10$ is true.

Reflect

6. In Example 3 above, the graphs also intersect to the left of the y-axis. Why is that point irrelevant to the problem?
That point is irrelevant to the problem since negative time has no meaning in this problem.

Your Turn

7. **Nature** An egg falls from a nest in a tree 25 feet off the ground and lands on a potted plant that is 20 feet below the nest. The function $h(t) = -16t^2 + 25$ gives the height in feet of the egg as it drops, where t represents time. When will the egg land on the plant?

$-16x^2 + 25 = 5$

$y = -16x^2 + 25$ and $y = 5$.

The egg will hit the plant after about 1.12 seconds.

QUESTIONING STRATEGIES

When finding the time it takes for an object to fall to the ground, why is only the positive zero of the function used as an answer? **Since the value for time will always be a positive number, the negative zero of the function can be ignored as a solution.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Technology

MP.5 Students who have the minimum and maximum x- and y-values set incorrectly on their graphing calculators may not be able to see the point of intersection when they graph two functions. Discuss how to change the dimensions of the graph by accessing the **WINDOW** menu.

EXPLAIN 4

Interpreting a Quadratic Model

AVOID COMMON ERRORS

When viewing graphs of quadratic functions modeling height, students may believe that the shape of the graph represents the path an object takes while in the air. Remind students that while the y-axis represents the height of the object, the x-axis does not show distance, but rather time.

QUESTIONING STRATEGIES

? For quadratic functions that model the height of a thrown object, how can you use the zeros of the function to determine when the object is at its maximum height? Explain. **Since the axis of symmetry for a quadratic function lies midway between the two zeros of the function, the maximum height is reached at a time halfway between the two zeros.**

? For a quadratic function modeling the height of a thrown object, when one of the zeros of the function is 0, what must be true about the object? **The object must have been thrown from ground level, so that the height of the object equals 0 at time 0.**

Explain 4 Interpreting a Quadratic Model

The solutions of a quadratic equation can be used to find other information about the situation modeled by the related function.

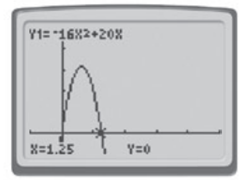
Example 4 **Use the given quadratic function model to answer questions about the situation it models.**

Ⓐ **Nature** A dolphin jumps out of the water. The quadratic function $h(t) = -16t^2 + 20t$ models the dolphin's height above the water in feet after t seconds. How long is the dolphin out of the water?

Use the level of the water as a height of 0 feet. $h(0) = 0$, so the dolphin leaves the water at $t = 0$. When the dolphin reenters the water again, its height is 0 feet.

Solve $0 = -16t^2 + 20t$ to find the time when the dolphin reenters the water.

Graph the function on a graphing calculator, and find the other zero that occurs at $x > 0$.



The zeros appear to be 0 and 1.25.

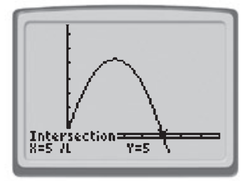
Check $x = 1.25$.

$-16(1.25)^2 + 20(1.25) = 0$ so 1.25 is a solution.

The dolphin is out of the water for 1.25 seconds.

Ⓑ **Sports** A baseball coach uses a pitching machine to simulate pop flies during practice. The quadratic function $h(t) = -16t^2 + 80t + 5$ models the height in feet of the baseball after t seconds. The ball leaves the pitching machine and is caught at a height of 5 feet. How long is the baseball in the air?

To find when the ball is caught at a height of 5 feet, you need to solve $5 = -16t^2 + 80t + 5$.



Graph Y1 = $\boxed{-16x^2 + 80x + 5}$ and Y2 = $\boxed{5}$, and use the intersection feature to find x-values when $y = 5$.

From the graph, it appears that the ball is 5 feet above the ground when $y = \boxed{0}$ or $y = \boxed{5}$.

Therefore, the ball is in the air for $\boxed{5} - 0 = \boxed{5}$ seconds.

Your Turn

8. **Nature** The quadratic function $y = -16x^2 + 5x$ models the height, in feet, of a flying fish above the water after x seconds. How long is the flying fish out of the water?

The graph of $y = -16x^2 + 5x$ shows a zero at about 0.3125.

The fish is out of the water for 0.3125 second.

Elaborate

9. How is graphing quadratic functions in standard form similar to using zeros to solve quadratic equations graphically?

If there are two solutions for a quadratic function, the reflection of the point representing one solution across the axis of symmetry will be the other point, which represents the other solution. Both methods use the value of the function at 0 to find the second point.

10. How can graphing calculators be used to solve real-world problems represented by quadratic equations?

Graphing calculators can be used to solve real-world quadratic equations by writing the equation and then finding either an intersection or the zeros of the equation's graph.

11. **Essential Question Check-In** How can you use the graph of a quadratic function to solve a related quadratic equation by way of intersection?

You can graph the two sides of the equation as two functions and find their points of intersection. The *x*-coordinate or coordinates of the intersection(s) will be the solution to the quadratic equation.

ELABORATE

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Discuss with students what kinds of real-world problems can be solved by identifying the zeros of a quadratic function. Students should understand that the zeros of a quadratic function will not be the solution for every real-world problem.

SUMMARIZE THE LESSON

How do you solve equations graphically using zeros and points of intersection? **To solve quadratic equations graphically using zeros, rewrite the equation so one side is equal to zero, then graph the other side of the equation and identify the zeros. To solve quadratic equations graphically using points of intersection, rewrite the equation in the form $ax^2 + bx = c$, graph both sides of the equation, then find the *x*-values of the points of intersection.**